OLORUNTOBA OLAJUBU

15/ENG05/019

Mechatronics

Assignment 4

1) $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$

$w^n = (1-x^2)\cancel{\frac{d^2y}{}}\, y''$

$v = (1-x^2)$ $\quad v' = -2x$ $\quad v'' = -2$ $\quad v''' = 0$

$u = y''$ $\quad u^n = y^{n+2}$

$w^n = u^n v + n u^{n-1} v' + \frac{n(n-1)u^{n-2}v''}{2} = 0$

$= y^{n+2}(1+x^2) + n y^{n-1}(-2x) \mp n(n-1)y^n$

$= (1-x^2)y^{n+2} - 2xny^{n-1} \mp n(n-1)y^n$

$w = -2xy'$

$v = -2x$ $\quad v' = -2$ $\quad v'' = 0$

$u = y'$ $\quad u^n = y^{n+1}$

$w^n = u^n v + n u^{n-1} v' + 0$

$= y^{n+1}(-2x) + n y^n(-2)$

$= -2x y^{n+1} - 2ny^n$

$w = 2y$

$v = 2$ $\quad v' = 0$

$u = y$ $\quad u^n = y^n$

$w^n = u^n v_0 + 0$

$= 2y^n$

$w = (1-x^2)y^{n+2} - 2xny^{n+1} \mp n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= (1-x^2)y^{n+2} + 2xy^{n+1}(-n-1) + y^n(-n^2-2n+2)$

at $x = 0$

$= y^{n+2} + y^n(n^2 - \cancel{2}n + 2) = 0$

$y^{n+2} = -y^n(n^2 - \cancel{2}n + 2)$

$n = 0$

$y^2 = -2y^0$

$n = 1$

$y^3 = -y^1[0] = 0$

$n = 2$

$y^4 = -y^2[-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3$

$y^5 = -y^3[-10] = 10y^3 = 0$

$n = 4$

$y^6 = -y^4[-18] = 18y^4 = 18\times(-8y^0) = -144y^0$

$n = 5$

$y^7 = -y^5[-28] = 28[y^5] = 0$

$y = y^0 + xy^1 + \frac{x^2}{2!}y^2 + \frac{x^3}{3!}y^3 + \frac{x^4}{4!}y^4 + \frac{x^5}{5!}y^5 + \frac{x^6}{6!}y^6 + \frac{x^7}{7!}y^7 \dots$

$y = y^0 + xy^1 + \frac{x^2}{2!}(-2y^0) + 0 + \frac{x^4}{4!}(-8y^0) + 0 + \frac{x^6}{6!}(-144y^0) + 0$

$y = y^0 + xy^1 - x^2y^0 - \frac{x^4}{3}y^0 - \frac{x^6}{5}y^0$

$y = y^0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y^1x$

2) (i) $L[3e^{-4t} - 5e^{4t}] = \frac{3}{s+4} - \frac{5}{s-4}$

(ii) $L[\sin 4t + \cos 4t] = \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

(iii) $L[t^3 + 2t^2 - t + 4] = \frac{3!}{s^{3+1}} + \frac{2(2!)}{s^{2+1}} - \frac{1}{s^{1+1}} + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

(iv) $L[e^{-2t}\cos 5t] = \frac{s+2}{(s+2)^2 + 5^2} = \frac{s+2}{(s+2)^2 + 25}$

(v) $L[t \sin 3t] = -\frac{d}{ds}\left[\frac{3}{s^2+9}\right]$

Let $u = 3$ $\quad \frac{du}{ds} = 0$ $\qquad v = s^2+9$ $\quad \frac{dv}{ds} = 2s$

$= -\frac{(s^2+9)\,0 - 3(2s)}{(s^2+9)^2}$

$= \frac{+6s}{(s^2+9)^2}$

(vi) $\frac{e^{-t} - e^{-2t}}{t}$

(vii) $L[e^{4t}\cos 2t] = \frac{s-4}{(s-4)^2+4}$

(viii) $L[t\sin 2t] = \cancel{\frac{-d}{ds}\left[\frac{2}{s^2+16}\right]}\ \frac{-d}{ds}\left[\frac{2}{s^2+4}\right]$

Let $u = 2$; $\frac{du}{ds} = 0$ $\quad v = s^2+4$; $\frac{dv}{ds} = 2s$

$\cancel{= -\frac{(s^2+16)\,0 - 8s}{(s^2+4)^2}} \quad \cancel{= \frac{-8s}{(s^2+16)^2}}$

$\cancel{= \frac{4-s^2}{(s^2+4)^2}} = \cancel{\frac{s^2-4}{(s^2+4)^2}}$

$= -\frac{0-4s}{(s^2+4)^2} = \frac{4s}{(s^2+4)^2}$

(ix) $L[t^3+4t^2+5] = \frac{3!}{s^{3+1}} + \frac{4(2!)}{s^{2+1}} + \frac{5}{s}$

$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$

(x) $L[e^{3t}(t^2+4)] = \frac{2!}{(s-3)^{2+1}} + \frac{4}{(s-3)}$

$= \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

(xi) $L[t^2\cos t] = \frac{d^2}{ds^2}\left[\frac{s}{s^2+1}\right]$

$u = s$; $\frac{du}{ds} = 1$ $\quad v = s^2+1$ $\quad \frac{dv}{ds} = 2s$

$\cancel{= \frac{d}{ds}\left[\frac{(s+1)-s}{(s+1)^2}\right]}$

$\cancel{= \frac{d}{ds}\left[\frac{1}{(s+1)^2}\right]}$ $\cancel{u = 1 \quad \frac{du}{ds} = 0 \quad v = (s+1)^2 \quad \frac{dv}{ds} = 2(s+1)}$

$\cancel{= \frac{(s+1)^2\cdot 0 - 2s+1}{(s+1)^4}}$

$\cancel{= \frac{-2s+1}{(s+1)^4}}$

$= \frac{d}{ds}\left[\frac{(s^2+1)-2s^2}{(s^2+1)^2}\right] = \frac{d}{ds}\left[\frac{1-s^2}{(s^2+1)^2}\right]$

$u = 1-s^2$; $\frac{du}{ds} = -2s$ $\quad v = (s^2+1)^2$ $\quad \frac{dv}{ds} = 4s(s^2+1)$

$= \frac{(s^2+1)^2(-2s) - (1-s^2)(4s(s^2+1))}{(s^2+1)^4}$

$= \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)^4}$

$= \frac{-2s(s^2+1)^2 - 4s(s^2+1)(1-s^2)}{(s^2+1)^{24}}$

(5) (i) $\dfrac{s-5}{(s-3)(s-4)} = \dfrac{A}{(s-3)} + \dfrac{B}{(s-4)}$

For A let $s = 3$: $\dfrac{s-5}{s-4} = \dfrac{3-5}{3-4} = \dfrac{-2}{-1} = 2$

For B let $s = 4$: $\dfrac{s-5}{s-3} = \dfrac{4-5}{4-3} = \dfrac{-1}{1} = -1$

$\therefore \dfrac{s-5}{(s-3)(s-4)} = \dfrac{2}{(s-3)} - \dfrac{1}{(s-4)}$

$L^{-1}\left[\dfrac{s-5}{(s-3)(s-4)}\right] = 2e^{3t} - e^{4t}$

(ii) $\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{A}{s-2} + \dfrac{B}{(s-4)}$

For A let $s = 2$: $\dfrac{2s-6}{s-4} = \dfrac{4-6}{2-4} = \dfrac{-2}{-2} = 1$

For B let $s = 4$: $\dfrac{2s-6}{s-2} = \dfrac{8-6}{4-2} = \dfrac{2}{2} = 1$

$\dfrac{2s-6}{(s-2)(s-4)} = \dfrac{1}{s-2} + \dfrac{1}{s-4}$

$L^{-1}\left[\dfrac{2s-6}{(s-2)(s-4)}\right] = e^{2t} + e^{4t}$

(iii) $\dfrac{5s-8}{s(s-4)} = \dfrac{A}{s} + \dfrac{B}{s-4}$

For A let $s = 0$ = $\dfrac{5s-8}{(s-4)} = \dfrac{-8}{-4} = 2$

For A let $s = 4$ = $\dfrac{5s-8}{s} = \dfrac{20-8}{4} = \dfrac{12}{4} = 3$

$\therefore \dfrac{5s-8}{s(s-4)} = \dfrac{2}{s} + \dfrac{3}{s-4}$

$L^{-1}\left[\dfrac{5s-8}{s(s-4)}\right] = 2 + 3e^{4t}$

iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{(s-1)^2}$

$s^2-3s-4 = A(s-1)^2 + B(s-3(s-1)) + C(s-3)$

$= A(s^2-2s+1) + B(s^2-s-3s+3) + C(s-3)$

$= A(s^2-2s+1) + B(s^2-4s+3) + C(s-3)$

$s^2 = As^2 + Bs^2$ $\quad -3s = -2As - 4Bs + Cs$ $\quad -4 = A+3B-3C$

$1 = A+B$ $\quad -3 = -2A - 4B + C$

$A = B-1$

$-3 = -2(B+1) - 4B + C$ $\quad -4 = B+1+3B-3C$

$-3 = -2B - 2 - 4B + C$ $\quad -5 = 4B - 3C$

$-3+2 = -6B + C$

$-1 = -6B + C$

$-3 = -18B + 3C$

$-3 = -18B + 3C$

$-5 = +4B - 3C$

$-8 = -14B$

$B = \frac{-8}{-14} = \frac{4}{7}$

$A = B-1$

$A = \frac{4}{7} - 1$

$A = -\frac{3}{7}$

$-5 = 4\left(\frac{4}{7}\right) - 3C$

$-5 = \frac{16}{7} - 3C$

$3C = \frac{16}{7} + 5$

$3C = \frac{51}{7}$

$C = \frac{17}{7}$

$\therefore \frac{s^2-3s-4}{(s-3)(s-1)^2} = \frac{-3}{7(s-3)} + \frac{4}{7(s-1)} + \frac{17}{7(s-1)^2}$

$L^{-1}\left[\frac{s^2-3s-4}{(s-3)(s-1)^2}\right] = \frac{-3}{7}e^{3t} + \frac{4}{7}e^{t} + \frac{17}{7}te^{t}$

v) $\frac{s-5}{s^2+4s+20} = \frac{s-5}{s+4s+4+16}$

$= \frac{s+2-7}{(s+2)^2-4^2} = \frac{s+2}{(s+2)^2-4^2} - \frac{7}{(s+2)^2-4^2}$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{8}{4}\left(\frac{4}{(s+2)^2+4^2}\right)$$

$$L^{-1}\left[\frac{s+5}{s^2+4s+20}\right] = e^{-2t}\cos 4t - \frac{7}{4}e^{-2t}\sin 4t$$