EFREFUL EDWARD SAMUEL
15/ENG01/026
CHEMICAL ENGINEERING

2. $dy/dt + 3y = e^{-2t}$ given that at $t=0, y=2$

$\mathcal{L}\{dy/dt\} + 3\mathcal{L}\{y\} = \mathcal{L}\{e^{-2t}\}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As+3A+Bs+2B$

$A+B = 2 \quad *3$

$3A+2B = 5 \quad *1$

$3A+3B = 6$

$3A+2B = 5$

$B = 1$

From eqn (1)

$$\frac{2s+3}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{s+2} + \frac{1}{s+3}\right\}$$

$$y = e^{-2t} + e^{-3t}$$

From eqn (2)

$$5A = 12 - B$$

$$B = -1/3$$

$$3A = 3 - 6$$

$$3A = 3 - B$$

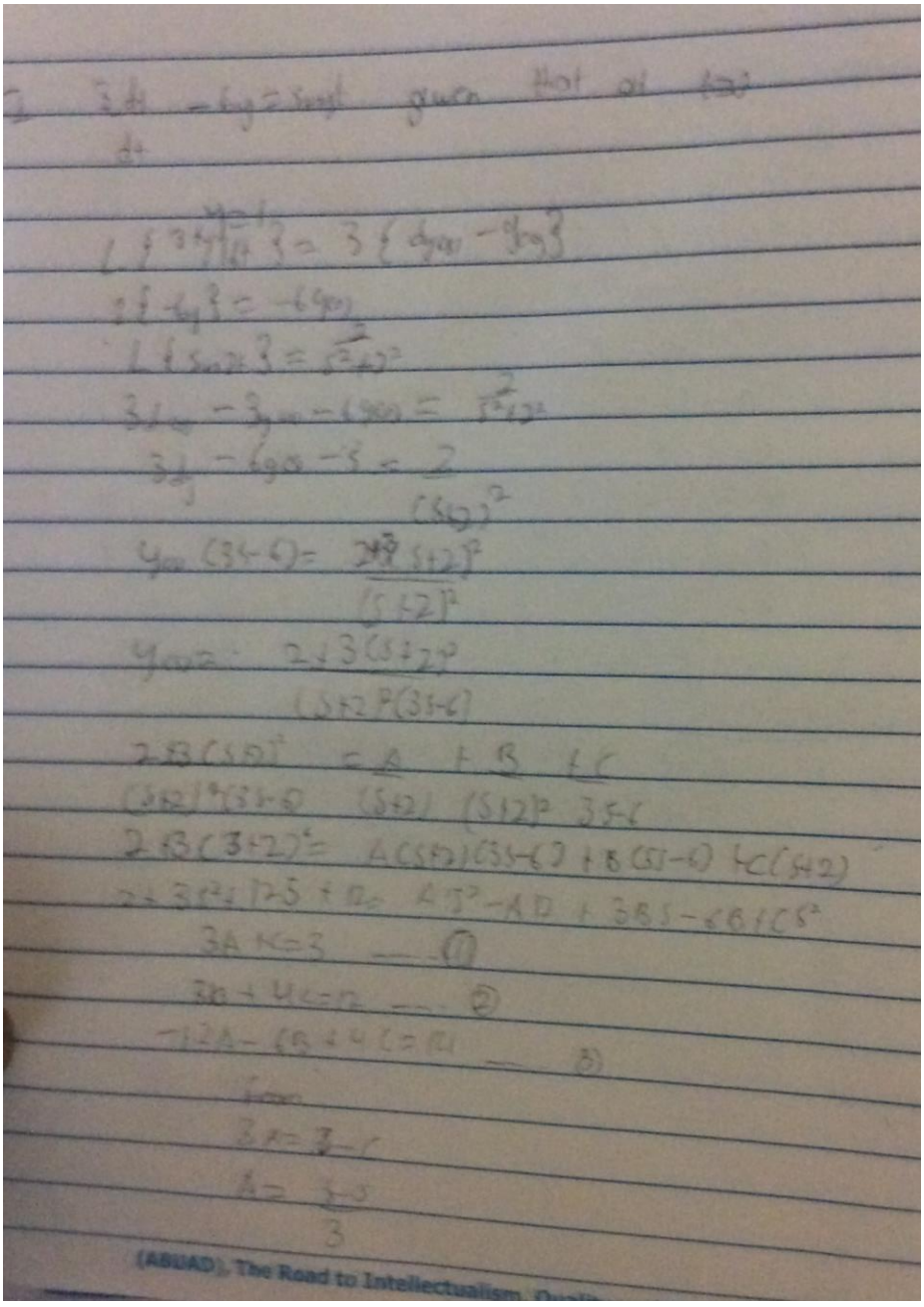
$$A = -1/2$$

$$\frac{2s+3}{(s+2)^2(3s-6)} = \frac{-1/2}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(2s-6)}$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1/2}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{2s-6}\right\}$$

$$y = -\frac{1}{2}e^{-2t} - \frac{1}{3}te^{-2t} + \frac{13}{12}e^{3t}$$

$$y = y_1(e^{-2t} + 4te^{-2t} - 3e^{3t})$$



$5B + 9C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B$

$-12 + 4C - 6B + 4C = 19$

$-6B + 8C = 28$

$-18B - 24C = -72$

$-18B + 24C = 84$

$48C = -56$

$C = 13/4$

$\frac{8+2s}{} = A(s-4) + Bs$

$3A = 3 - C \quad 2 + B = 2$

$3A = 3 - \frac{13}{4} \quad -4A = 8$

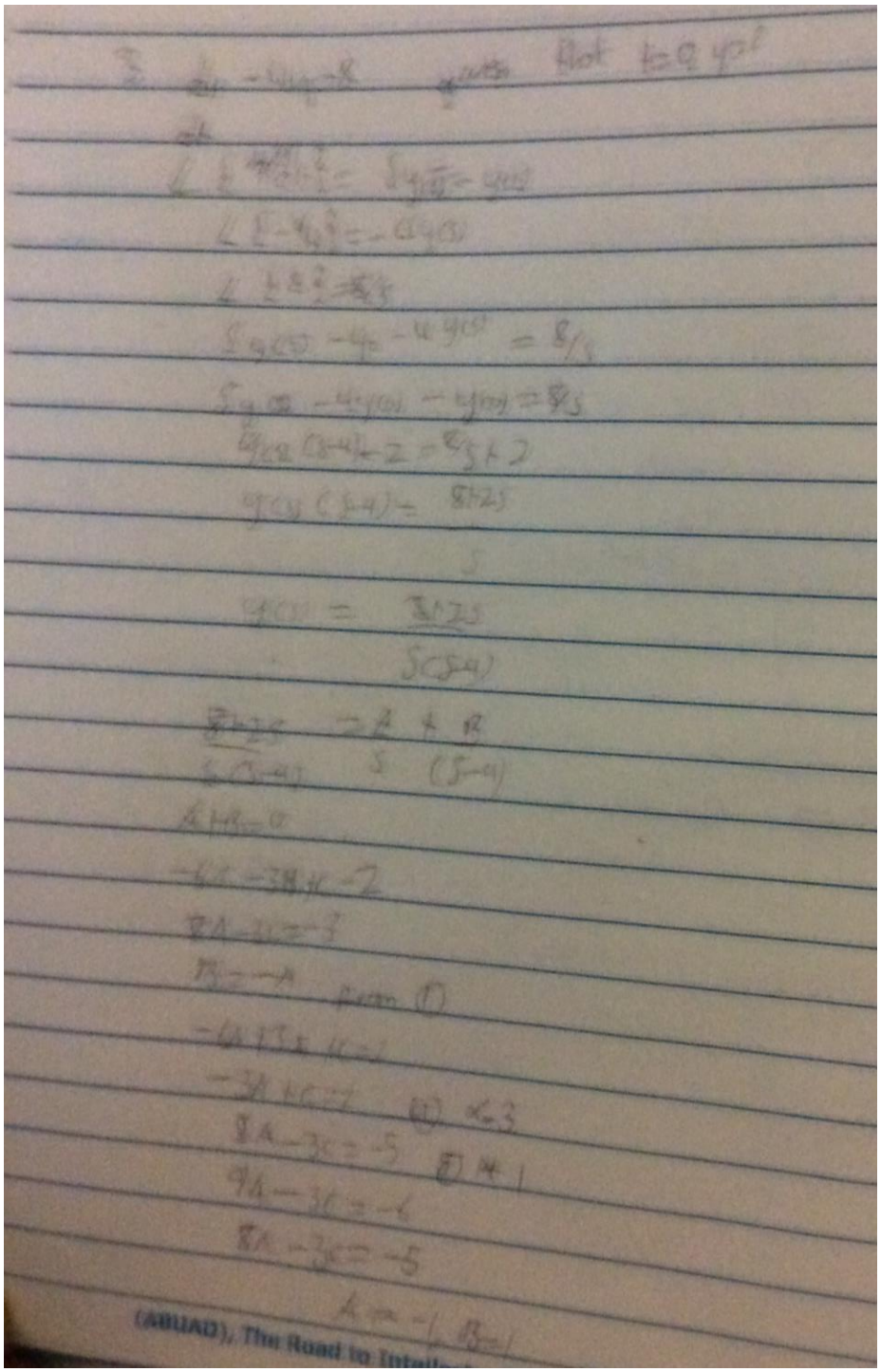
$A = -1/12 \quad A = -2$

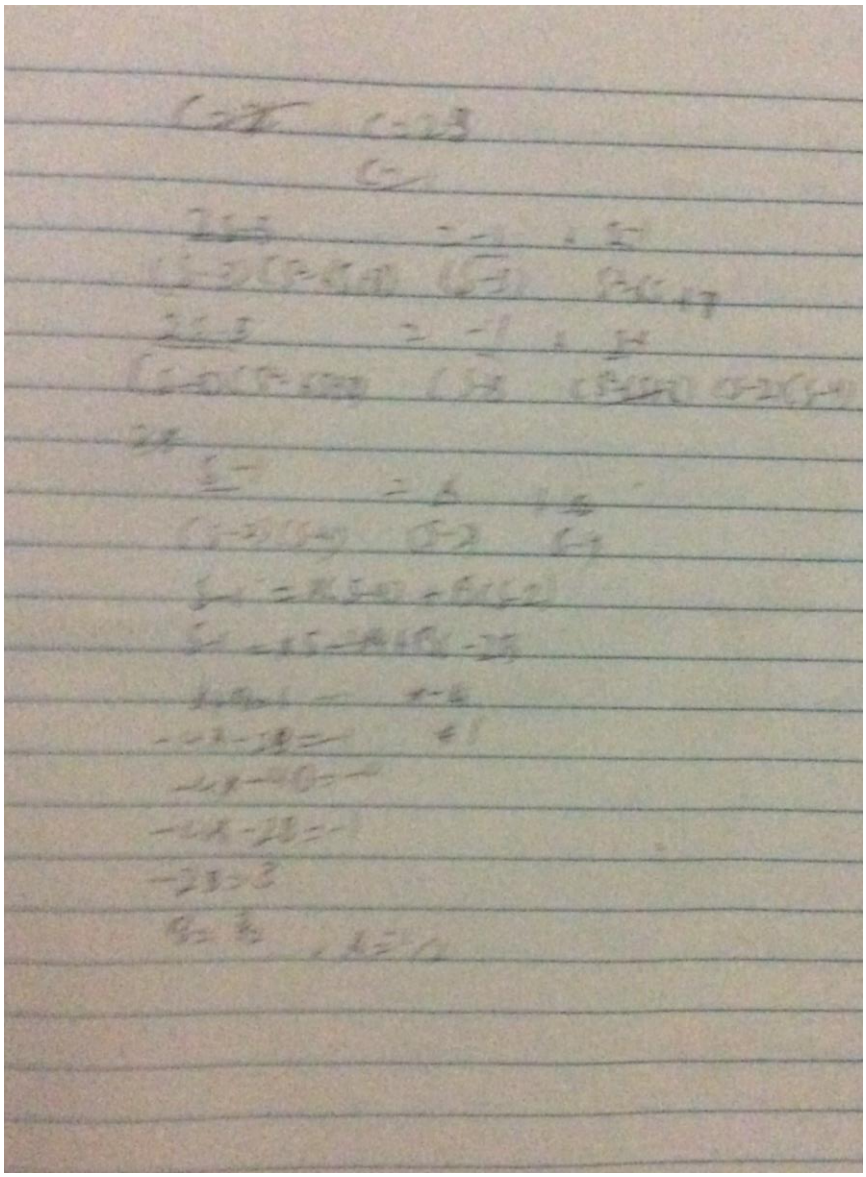
$3A = \quad B = 2 + 2 \quad B = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L\{y(t)\} = L^{-1}\{-2/s + 4/(s-4)\}$

$y = -2 + 4e^{4t}$





4 $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^t$ given that at $y=2, y'=0$

$\mathcal{L}\{d^2y/dt^2\} = s^2Y(s) - sy(0) - y'(0)$

$\mathcal{L}\{-2dy/dt\} = -2sY(s) + 2y(0)$

$\mathcal{L}\{5y\} = 5Y(s)$

$\mathcal{L}\{e^t\} = \frac{1}{s-1}$

$s^2Y(s) - 2s - 0 - 2sY(s) + 4 + 5Y(s) = \frac{1}{s-1}$

$s^2Y(s) - 2sY(s) + 5Y(s) - 2s + 4 = \frac{1}{s-1}$

$Y(s)(s^2 - 2s + 5) = \frac{1}{s-1} + 2s - 4$

$Y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-1)}$

$Y(s) = \frac{1 + 2s^2 - 5s + 6}{(s-1)(s^2 - 2s + 5)}$

$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{(s-4)}\right)$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{-1}{(s-3)} - \frac{1/2}{(s-2)} + \frac{3/2}{(s-4)}\right\}$

$y = -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$

$y = \frac{1}{2}(2e^{3t} + e^{2t} - 3e^{4t})$