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15/ENG01/060 ENG 387

CHEMICAL ENGINEERING

ASSIGNMENT V

(1) $dy/dt + 3y = e^{-2t}$

at $t=0$, $y=2$

$s\,Y(s) - y_0 + 3\,(Y(s)) = 1/s+2$

$s\,Y(s) - 2 + 3\,Y(s) = 1/s+2$

$Y(s)\,(s+3) = 1/s+2 + 2$

$Y(s)\,(s+3) = \frac{1+2(s+2)}{s+2}$

$Y(s)\,(s+3) = \frac{1+2s+4}{s+2}$

$Y(s)\,(s+3) = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

when $s=-2$

$2(-2)+5 = A(-2+3)$

$-4+5 = A$

$A = 1$

when $s=-3$

$2(-3)+5 = B(-3+2)$

$-6+5 = -B$

$-1 = -B$

$B = 1$

$Y(s) = 1/s+2 + 1/s+3$

$y = e^{-2t} + e^{-3t}$

(2) $3\,dy/dt - 6y = \sin 2t$

at $t=0$, $y=1$

$L\left(\frac{3dy}{dt}\right) = 3s(Y(s)) - 3y_0$

$L(-6y) = -6Y(s)$

$L(\sin 2t) = 2/s^2+4$

$3s(Ys) - 3\times 1 - 6Y(s) = 2/s^2+4$

at $t=0,\ y=1$

$3sy(s) - 3\times 1 - 6y(s) = 2/s^2+4$

$3sy(s) - 3 - 6y(s) = 2/s^2+4$

$y(s)\ (3s-6) = 2/s^2+4 + 3$

$ys(3s-6) = 3s^2+4\ /s^2+4$

$y(s) = 3s^2+14/(s^2+4)(3s-6)$

using partial fraction

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{s^2+4} + \frac{C}{3s-6}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{s^2+4} + \frac{C}{3s-6}$$

$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$

$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$

$A = \frac{-1}{6}$, $B = -1/12$, $C = \frac{13}{4}$

$$Y(s) = \frac{\frac{-1}{6} - (1/12)s}{s^2+4} + \frac{\frac{13}{4}}{s^2-6}$$

$$= \frac{-1/6}{s^2+4} - \frac{1/12\ s}{s^2+4} + 13/4/3s-6$$

$= -1/6 - 1/s^2+2^2 - 1/12 \cdot s/s^2+2^2 + 13/4 \cdot 1/3(s-2)$

$= -1/6 \cdot 1/2(2/s^2+2^2) - 1/12(s/s^2+2^2) + 13/12(1/s-2)$

$y(t) = -1/12 \sin 2t - 1/12 \cos 2t + \frac{13}{2} e^{2t}$

$y_t = 1/12(-\sin 2t - \cos 2t + 13e^{2t})$

$y_t = \frac{1}{12}(13e^{2t} - \cos 2t - \sin 2t)$

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(3) $\frac{dy}{dt} - 4y = 8$

$t = 0, y = 2$

$y' - 4y = 8$

$sy(s) - y(0) - 4(y_s) = 8/s$

Applying the condition

$y(s)(s-4) = \frac{8}{s} + \frac{2}{1} = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

$\left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

(4) $d^2y/dx^2 - 2\,dy/dx + 5y = e^{2t}$

$t = 0, y = 2, y'(0) = 1$

$y'' - 2y' + 5y = e^{2t}$

$\left(s^2y(s) - sy(0) - y'(0)\right) - 2\left(sy(s) - y(0)\right) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$y(s)(s^2 - 2s - 3) = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$

using partial fraction

$\frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs + C}{s^2 - 2s + 5}$

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$$2s^2-7s+7 = A(s^2-2s+5)+(Bs+c)(s-2)$$

$$2s^2-7s+7 = As^2-2As+5A+Bs^2-2Bs+cs-2c$$

$$2 = A+B$$

$$A = 1/5 \qquad y(s) = \frac{\frac{1}{5}}{s-2} + \frac{\frac{9}{5}s-3}{s^2-2s+5}$$

$$2 = 1/5+B$$

$$B = 9/5$$

$$\cancel{yt = 1/5\,e}$$

$$7 = 5A-2c$$

$$7 = 5(1/5)-2c$$

$$7-1 = -2c$$

$$c = -3$$

$$yt = \frac{1}{5}\left(e^{2t}+e^{-t}(9\cos 2t-10\sin 2t)\right)$$