Name: KELLY MIRACLE-U.
Matric No: 15/ENG07/027
Dept:- petroleum Eng.

1) $dy/dt + 3y = e^{-2t}$

at $t=0$, $y=2$

$sY(s) - y_0 + 3[Y(s)] = 1/s+2$

$sY(s) - 2 + 3Y(s) = 1/s+2$

$Y(s)(s+3) = \frac{1}{s+2} + 2$

$Y(s)(s+3) = \frac{1+2(s+2)}{s+2}$

$Y(s)(s+3) = \frac{2s+5}{s+2}$

$Y(s)(s+3) = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$2(-2)+5 = A(-2+3)$

$-4+5 = A$

$A = 1$

When $s=-3$

$2(-3)+5 = B(-3+2)$

$-6+5 = -B$

$-1 = -B$

$B = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y = e^{-2t} + e^{-3t}$

(2) $3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0, y=1$

$3(sY(s) - y_0) - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 3y_0 - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)(3s-6) - 3 = \frac{2}{s^2+4}$

$Y(s)(3s-6) - 3 = \frac{2}{s^2+4}$

$Y(s) = \left(\frac{2}{s^2+4} + 3\right) \div (3s-6)$

$Y(s) = \frac{2+3(s^2+4)}{(s^2+4)(3s-6)}$

$Y(s) = \frac{2+3s^2+12}{(s^2+4)(3s-6)}$

$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$

$3s^2+14 = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$

$3s^2+14 = As + B(3s-6) + C(s^2+4)$

$3s^2+14 = 3As^2 - 6As + 3Bs - 6B + Cs^2 + 4C$

$3A + C = 3$ — (i)

$-6A + 3B = 0$ — (ii)

$-6B + 4C = 14$ — (iii)

$C = 3 - 3A$ — (iv)

$-6B + 4(3 - 3A) = 14$

$-6B + 12 - 12A = 14$

(2) $-6A + 3B = 6$

$3B = +6A$

$B = +2A$

$-6(2A) - 12A = 2$

$+12A + 12A = 2$

$24A = 2$

$A = +1/12$

$B = +2 \times -1/12$

$B = -1/6$

$C = 3 - 3(-1/12)$

$C = 3 + 3/12$

$\frac{3(12)+3}{12}$

$C = \frac{36+3}{12} \quad C = \frac{39}{12} = 13/4$

$$Y(s) = \frac{\frac{s^2}{12} - 1/6}{s^2+4} + 13/4\left(\frac{1}{s-12}\right)$$

$$Y(s) = \frac{s}{12}\left(\frac{1}{s^2+2^2}\right) - 1/6\left(\frac{1}{s^2+2^2}\right) + \frac{13}{12}\left(\frac{1}{s-12}\right)$$

$$= \frac{1}{12}\left(\frac{s}{s^2+2^2}\right) - \frac{1}{6}\left(\frac{1}{s^2+2^2} \times 1/2 \times \frac{2}{1}\right) + \frac{13}{12}\left(\frac{1}{s-2}\right)$$

$$= \frac{1}{12}\left(\frac{s}{s^2+2^2}\right) - \frac{1}{12}\left(\frac{2}{s^2+2^2}\right) + \frac{13}{12}\left(\frac{1}{s-2}\right)$$

$$= 1/12\cos 2t - 1/2\sin 2t + \frac{13}{12}e^{2t}$$

3) $dy/dt - 4y = 8$ @ $t = 0, y = 2$ $y(0) = 2$

$y' - 4y$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)[s-4] = 8/s + 2/1 = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s+4)} = \frac{A}{s} + \frac{B}{s-4}$

$A: \left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

$B: \left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$

$y(s) = -2/s + 4/s - 4$

$y(t) = -2 + 4e^{4t}$

7) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$s^2Y(s) - sy_0 - y_0' - 2(sY(s) - y_0) + 5(Y(s)) = \frac{1}{s-2}$

$s^2Y(s) - 5y_0 - y_0' - 2sY(s) + 2y_0 + 5Y(s) = \frac{1}{s-2}$

$Y(s)(s^2 - 2s + 5) - 2s - 1 + 4 = \frac{1}{s-2}$

$Y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$

$Y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + \frac{2s-3}{1}$

$y(s)(s^2 - 2s + 5) = 1$

$y(s)(s^2 - 2s + 5) = \frac{1 + 2s - 3(s-2)}{(s-2)}$

$y(s)(s^2 - 2s + 5) = \frac{1 + 2s^2 - 9s - 3s + 6}{(s-2)}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 7s + 7}{(s-2)}$

$y(s) = \frac{2s^2 + 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$

$A: \left.\frac{2s^2 - 7s + 7}{s^2 - 2s - 5}\right|_{s=-2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s - 2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2 = A + B$

$2 = \frac{1}{5} + B$

$B = \frac{9}{5}$

$7 = 5A - 2C$

$7 = 5(\frac{1}{5}) - 2C$

$7 - 1 = 2C$

(f)

$C = -3$

$$g(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9s/5 - 3}{s^2 - 2s + 8} - \frac{1}{8}\cdot\frac{1}{s-2} + \frac{9s/5}{(s+1)^2+4} - \frac{3}{(s+$$

$$g(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s-1+1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$g(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{(s+1)}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$g(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{(s+1)}{(s+1)^2+2^2} - \frac{1}{(s+1)^2+4} - \frac{3}{(s+1)^2+2^2}$$

$$g(s) = \frac{1}{s}\cdot\frac{1}{s-2} + \frac{9}{5}\cdot\frac{s+1}{(s+1)^2+2^2} - \frac{4}{(s+1)^2+2^2}\cdot\frac{2}{2}$$

$$g(t) = \frac{1}{8}e^{2t} + \frac{9}{8}e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

$$g(t) = \frac{1}{8}\left[e^{2t} + 9e^{-t}\cos 2t - 16e^{-t}\sin 2t\right]$$

5) $d^2y/dx^2 - 6\,dy/dt + 8y = e^{3t}$ at $t=0$ $y=0$ $y'=2$

$y'' - 6y' + 8y = e^{3t}$

$s^2y(s) - sy(0) - y'(0) - 6[sy(s) - y(0)] + 8y(s) = 1/s - 3$

$s^2y(s) - 2 - 6sy(s) + 8y(s) = 1/s - 3$

$y(s)[s^2 - 6s + 8] = 1/s - 3 + 2/1 = \frac{1 + 2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$

$y(s) = \frac{2s-5}{(s-3)(s-6s+5)} = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} - \frac{C}{s-4}$

$A: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$

$B: \left.\frac{2s-5}{(s-2)(s-4)}\right|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = -1/2$

$C: \left.\frac{2s-5}{(s-3)(s-2)}\right|_{s=4} = \frac{2(4)-5}{(4-3)(4-2)} = 3/2$

$y(s) = \frac{-1}{s-3} - 1/2 - \frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}$

$y(t) = -e^{3t} - \frac{1}{2}e^{2t} + 3/2\,e^{4t}$