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15/ENG06/030
Mechanical Engineering

Assignment 5

$\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0$, $y=2$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{3y\} = 3y$

$L\{e^{-2t}\} = \frac{1}{s+2}$

$sy(s) - s(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = 3A + As + Bs + 2B$

$A+B = 2 \times 3$

$3A+2B = 5 \times 1$

$3A + 3B = 6$

$3A + 2B = 5$

$B = 1$

from eqn ①

$A + 1 = 2$

$A = 2 - 1 = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{(s+2)} + \frac{1}{(s+3)}\right\}$

$= e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$ given that $t=0,\ y=1$

$L\{3\,dy/dt\} = 3\{sy(s) - y(0)\}$

$L\{-6y\} = -6y(s)$

$L\{\sin 2t\} = \frac{2}{s^2+2^2}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+2^2}$

$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$

$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$

$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+12s+12 = As^2 - A12 + 3Bs + 3As - 6B + Cs^2 + 4Cs + 4C$

$3A + C = 3$ — 1

$-12A - 6B + 4C = 14$ — 2

$3B + 4C = 12$ — 3

From 1

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-B+4c-6B+4c=14$ ----

$-6B+8c=26$

$-18B-24c=-72$

$-18B+24c=78$

$-48c=150$

$c=\frac{+25}{8}$

from eqn ②

$3B=12-4\left(\frac{25}{8}\right)$

$B=1/6$

from ①

$3A=3-C$

$A=\frac{3-(25/8)}{3}$

$A=-1/24$

$$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)}=\frac{-1/24}{(s+2)}-\frac{1/6}{(s+2)^2}+\frac{25/8}{(3s-6)}$$

$$L^{-1}\{y_{(s)}\}=L^{-1}\left\{\frac{-1/24}{(s+2)}-\frac{1/6}{(s+2)^2}+\frac{25/8}{3s-6}\right\}$$

$$y=\frac{-1}{24}e^{-2t}-\frac{1}{6}te^{-2t}+\frac{25}{24}e^{2t}$$

$$y=\frac{-1}{6}\left(\frac{1}{4}e^{-2t}-te^{-2t}+\frac{25}{4}e^{2t}\right\}$$

) $\frac{dy}{dt}-4y=8$ given that $t=0$, $y=2$

$L\{dy/dt\}=sy_{(s)}-y_{(0)}$

$L\{-4y\}=-4y_{(s)}$

$L\{8\}=8/s$

$sy_{(s)}-y_{(0)}-4y_{(s)}=8/s$

$sy_{(s)}-4y_{(s)}-y_{(0)}=8/s$

3) $y_{(s)}(s-4) - 2 = \frac{8}{s} + 2$

$y_{(s)}(s-4) = \frac{8+2s}{s}$

$y_{(s)} = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = A(s-4) + B(s)$

$A+B=2$

$-4A=8$

$A=-2$

$B=2+2=4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\{y_{(s)}\} = L^{-1}\left\{\frac{-2}{s} + \frac{4}{(s-4)}\right\}$

$y = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given that $t=0$, $y=2, 2'$

$L\left\{\frac{d^2y}{dt^2}\right\} - L\left\{2\frac{dy}{dt}\right\} + L\{5y\} = L\{e^{2t}\}$

$s^2y_{(s)} - sy_{(0)} - y'_{(0)} - 2sy_{(s)} + 2y_{(0)} + 5y_{(s)} = \frac{1}{s-2}$

$s^2y_{(s)} - 2sy_{(s)} + 5y_{(s)} - 2s - 1 + 4 = \frac{1}{s-2}$

$y_{(s)}(s^2-2s+5) = \frac{1}{s-2} + 2s - 3$

$y_{(s)}(s^2-2s+5) = \frac{(2s-3)(s-2)}{(s-2)}$

$y_{(s)} = \frac{1 + 2s^2 - 5 + 6}{(s-2)(s^2-2s+5)}$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}$$

$2s^2-7s+7 = A(s^2-2s+5)+Bs+C(s-2)$

$2s^2-7s+7 = As^2-2As+5A-Bs^2-2Bs+Cs-2C$

$A+B=2$ — ①

$-2A-2Bs+C=7$ — ②

$5A-2C=7$

from ①

$B=2-A$

from ②

$-2A-2(2-A)+C=7$

$-2A-4+2A+C=-7$

$C=3$

from ③

$5A-2(-3)=7$

$A=7-6$

$A=1/5$

$A+B=2$

$\frac{1}{5}+B=2$

$B=2-\frac{1}{5}$

$B=\frac{9}{5}$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5\,s-3}{(s^2-2s+5)}$$

$$L^{-1}\{y(s)\} = L^{-1} = \left\{\frac{\frac{1}{5}}{(s-2)} + \frac{9/5\,s}{(s^2-2s+5)} - \frac{3}{(s^2-2s+5)}\right\}$$

$$y = L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2^2} - \frac{1\times 2/2}{(s-1)^2+2^2}\right) - \frac{3}{2}\left\{\frac{2}{(s-1)^2+2^2}\right\}$$

$$y = \frac{1}{5}e^{-2t} + \frac{9}{5}\left\{e^t\cos 2t + \frac{1}{2}e^t\sin 2t\right\} - \frac{3}{2}\left(e^t\sin 2t\right)$$

(5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ given that $t=0, y=0, y'=2$

$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y(s) - sy(0) - y'(0)$

$L\left\{-6\frac{dy}{dt}\right\} = -6sy(s) + 6y(0)$

$L\{8y\} = 8y(s)$

$L\{e^{3t}\} = \frac{1}{s-3}$

$s^2y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{(s-3)}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{Bs+C}{(s^2-6s+8)}$

$2s - 5 = A(s^2 - 6s + 8) + Bs + C(s-3)$

$2s - 5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$

$A + B = 0$

$-6A - 3B + C = 2$

$8A - 3C = -3$

$B = -A$ —— eqn (1)

$-6A + 3A + C = 2$

$-3A + C = 2$ —— (2)

$8A - 3C = -5$ —— (3)

$9A - 3C = -6$

$-$ $8A - 3C = -5$

$A = 1$

$\therefore A = 1$

$B = -A$

$B = -1$

from eqn ③

$c = 2 - 3$

$c = -1$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{1}{s-3} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{s-2} + \frac{B}{s-4}$$

$s-1 = A(s-4) + B(s-2)$

$s-1 = As - 4A + Bs - 2B$

$A + B = 1 \quad \text{—①} \quad [\times 4]$

$-4A - 2B = -1 \quad \text{—②} \quad [\times 1]$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = \frac{3}{2} \qquad A = \frac{-1}{2}$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{s-3} + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{s-4}\right)$$

$$L^{-1}[y_{(s)}] = L^{-1}\left\{\frac{-1}{s-3} - \frac{1/2}{s-2} + \frac{3/2}{s-4}\right\}$$

$y = e^{3t} - \frac{1}{2}e^{2t} - \frac{3}{2}e^{4t}$

$y = \frac{1}{2}\left\{2e^{3t} + e^{2t} - 3e^{4t}\right\}$