Amaefule Osmachi Charles
IS/ENG06/010
Mechanical Engineering.

i) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0, y=2$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

$$y(s) = \frac{1}{(s+2)(s+3)} + \frac{2}{(s+3)}$$

$$y(s) = \frac{1 + 2s + 4}{(s+2)(s+3)}$$

$$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$2s+5 = A(s+3) + B(s+2)$$

$$2s+5 = (A+B)s + 3A + 2B$$

$$A+B = 2$$

$$3A+2B = 5$$

$$A = 2 - B$$

$$3(2-B) + 2B = 5$$

$$6 - 3B + 2B = 5$$

$$6 - B = 5$$

$$B = 1$$

$$A = 2 - B$$

$$A = 2 - 1$$

$$A = 1$$

$$\therefore y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$y(t) = e^{-2t} + e^{-3t}$$

~~ii 3d~~

ii) $3\frac{dy}{dt} - 6y = \sin 2t$

$$\frac{dy}{dt} - 2y = \frac{\sin 2t}{3}$$

$$sy(s) - y(0) - 2y(s) = \frac{2}{3s^2 + 12}$$

$$y(s)(s-2) - 1 = \frac{2}{3s^2+12}$$

$$y(s)(s-2) = \frac{2}{3s^2+12} + 1$$

$$y(s) = \frac{2}{3s^2+12} \cdot + \frac{1}{s-2}$$

$$y(s) = \frac{2s-4+3s^2+12}{(3s^2+12)(s-2)}$$

$$y(s) = \frac{3s^2+2s+8}{(3s^2+12)(s-2)} = \frac{As+B}{3s^2+12} + \frac{C}{s-2}$$

$$3s^2+2s+8 = As^2-2As+Bs-2B+3Cs^2+12C$$

$A+3C = 3$ --- (1) $\times 4$

$B-2A = 2$

$12C-2B = 8$

$B = 2+2A$

$12C - 2(2+2A) = 8$

$12C - 4 - 4A = 8$

$-4A + 12C = 12$

$4A + 12C = 12$

$24C = 24$

$C = 1$

$A+3 = 3$

$A = 0$

$B = 2+0$

$B = 2$

$$\therefore y(s) = \frac{2}{3s^2+12} + \frac{1}{s-2}$$

$$y(s) = \frac{1}{3}\left(\frac{2}{s^2+2^2}\right) + \frac{1}{s-2}$$

$$y(t) = \frac{1}{3}\sin 2t + e^{2t}$$

Subject :

Form :

(iii) $\frac{dy}{dt} - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$y(s)(s-4) - 2 = \frac{8}{s}$

$y(s) = \frac{8}{s(s-4)} + \frac{2}{(s-4)}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$8+2s = As + Bs - 4A$

$A+B = 2$

$-4A = 8$

$A = -2$

$-2+B = 2$

$B = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

~~$y(s)$~~ $y(t) = -2 + 4e^{4t}$

(iv) $s^2y(s) - sy(0) - y'(0) - 2sy(s) + 2y(0) + 5y(s) = \frac{1}{s-2}$

$y(s)(s^2-2s+5) - 2s + 4 - 1 = \frac{1}{s-2}$

$y(s)(s^2-2s+5) - 2s + 3 = \frac{1}{s-2}$

$y(s) = \frac{1}{(s-2)(s^2-2s+5)} + \frac{2s-3}{s^2-2s+5}$

$y(s) = \frac{1 + 2s^2 - 4s - 3s + 6}{(s-2)(s^2-2s+5)}$

$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$

$2s^2-7s+7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2s^2-7s+7 = (A+B)s^2 + (-2A-2B+C)s + 5A - 2C$

$A+B=2$

$-2A-2B+C=-7$

$5A-2C=7$

~~$A=2-B$~~ $B=2-A$

$-2A-4+2A+C=-7$

$C=-3$

$5A+6=7$

$5A=1$

$A=1/5$

$B=2-1/5$

$B=9/5$

$\therefore Y(s)=\frac{1}{5(s-2)}+\frac{\frac{7}{5}s-3}{s^2-2s+5}$

$y(s)=\frac{1}{5}\left(\frac{1}{s-2}+\frac{9s-15}{s^2-s-5+1+4}\right)$

$y(s)=\frac{3}{5}\left(\frac{3}{s-2}+\frac{3s-5}{(s-1)^2+2^2}\right)$

$y(s)=\frac{3}{5}\left(\frac{3}{s-2}+\frac{3s-3-2}{(s-1)^2+2^2}\right)$

$y(s)=\frac{3}{5}\left(\frac{3}{s-2}+\frac{3(s-1)}{(s-1)^2+2^2}-\frac{2}{(s-1)^2+2^2}\right)$

$y(t)=\frac{3}{5}\left(3e^{+2t}+3e^{t}\cos 2t-e^{t}\sin 2t\right)$

$s^2y(s) - sy(0) - y'(0) - 6sy(s) + 6y(0) + 8y(s) = \frac{1}{s-3}$

$y(s)(s^2 - 6s + 8) - 2 = \frac{1}{s-3}$

$y(s) = \frac{1}{s-3(s^2-6s+8)} + \frac{2}{s^2-6s+8}$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8}$

$y(s) = \frac{2s-5}{(s-3)(s^2-4s-2s+8)}$

$y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$

$2s-5 = A(s-2)(s-4) + B(s-3)(s-4) + C(s-3)(s-2)$

let $s = 2$

$4-5 = B(-1)(-2)$

$-1 = 2B$

$B = -1/2$

let $s = 3$

$6-5 = -A$

$A = -1$

let $s = 4$

$8-5 = 2C$

$C = 3/2$

$\therefore y(s) = \frac{-1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)}$

$y(t) = \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t} - e^{3t}$

$y(t) = \frac{1}{2}(3e^{4t} - e^{2t} - 2e^{3t})$