

ASSIGNMENT 5

1] $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0, y=2$

$$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

$$sY(s) - 2 + 3Y(s) = \frac{1}{s+2}$$

$$sY(s) + 3Y(s) = \frac{1}{s+2} + 2$$

$$Y(s)(s+3) = \frac{1+2s+4}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$= \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

$$2s+5 = A(s+3) + B(s+2)$$

at $s = -2$

$$2(-2) + 5 = A(-2+3)$$

$$A = 1$$

at $s = -3$

$$2(-3) + 5 = B(-3+2)$$

$$B = 1$$

$$Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$\therefore y(t) = e^{-2t} + e^{-3t}$$

2] $\frac{dy}{dt} - 6y = \sin 2t$ at $t=0, y=1$

$$sY(s) - y(0) - 6Y(s) = \frac{2}{s^2+4}$$

$$sY(s) - 1 - 6Y(s) = \frac{2}{s^2+4}$$

$$sY(s) - 1 - 6Y(s) = \frac{2}{s^2+4}$$

$$2y'(t) - 6y(t) = \frac{2}{s^2+4} + 7$$

$$y'(s)(s-6) = \frac{2+7(s^2+4)}{s^2+4}$$

$$y(s) = \frac{7s^2+14}{(s^2+4)(s-6)}$$

$$\frac{7s^2+14}{(s^2+4)(s-6)} = \frac{A}{s^2+4} + \frac{B}{s-6} + \frac{C}{s-6} \quad C = 13/4$$

$$7s^2+14 = (A+B)s(s-6) + C(s^2+4)$$

$$7s^2+14 = 7As - (A+B)s - 6Bs + (s^2+4)C$$

$$7 = 7A + C$$

$$14 = -6A + C$$

$$A = -1/6$$

$$7A - 6B = 1$$

$$7A = 1$$

$$7A = 6(-1/6)$$

$$A = -1/6$$

$$y(s) = -\frac{1}{6} \left(\frac{1}{s^2+4} \right) - \frac{1}{12} \left(\frac{6}{s^2+4} \right) + \frac{13}{4} \left(\frac{1}{s-6} \right)$$

$$= -\frac{1}{6} \left(\frac{1}{s^2+2^2} \right) - \frac{1}{12} \left(\frac{6}{s^2+2^2} \right) + \frac{13}{12} \left(\frac{1}{s-2} \right)$$

$$= y(t) = -\frac{1}{12} \sin 2t - \frac{1}{12} (\cos 2t) + \frac{13}{12} e^{2t}$$

$$\} \frac{dy}{dt} - 4y = 8 \quad \text{at } t=0, y=2$$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = \frac{8}{s}$$

$$sy(s) - 4y(s) = \frac{8}{s} + 2$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2 = \frac{A(1-4) + B}{(1-4)}$$

$$y + 2 = A(1-4) + B$$

$$+ 2 = 0$$

$$2 = -4A$$

$$A = -\frac{1}{2}$$

$$+ 2 = 0$$

$$1 + 2(4) = 4B$$

$$4B = 9$$

$$B = \frac{9}{4}$$

$$y(t) = -\frac{1}{2} + \frac{9}{4}e^{4t}$$

$$\therefore y(t) = -\frac{1}{2} + \frac{9}{4}e^{4t}$$

$$y''/y' = 2 + y'/y = e^{1/x} \quad \text{at } x=0, \quad y=1, \quad y'=1$$

$$s^2 y(s) - s y(0) - y'(0) = 2 [s y(s) - y(0)] + s y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - s y(0) - y'(0) = 2 [s y(s) - y(0)] + s y(s) = \frac{1}{s-2}$$

$$y(s) (s^2 - 2s + 5) - 2(1) + 1 = \frac{1}{s-2}$$

$$y(s) (s^2 - 2s + 5) = \frac{1}{s-2} + 2(1) - 1$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$A = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} \Big|_{s=2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$2 = A + B$$

$$-7 = -2A + C$$

$$7 = 5A - 2C$$

$$7 = 5A - 2C$$

$$7 = 5(1/5) - 2C$$

$$y(t) = \frac{1}{5} \left(\frac{1}{s-2} \right) + \frac{1}{5} \left(\frac{s+1}{(s+1)^2+2^2} \right) - \frac{2}{5} \left(\frac{4}{(s+1)^2+2^2} \right)$$

$$y(t) = \frac{1}{5} \left(\frac{1}{s-2} \right) + \frac{1}{5} \left(\frac{s+1}{(s+1)^2+2^2} \right) - \frac{2}{5} \left(\frac{2}{(s+1)^2+2^2} \right)$$

$$= y(t) = \frac{1}{5} e^{2t} + \frac{1}{5} e^{-t} (\cos 2t - 2e^{-t} \sin 2t)$$

$$s^2 y(s) - (s^2 y(0) + s y'(0)) = e^{3s} \quad \text{at } t=0, \quad y=0, \quad y'=2$$

$$s^2 y(s) - s y(0) - y'(0) = (s y(s) + (y(0) + s y(s))) = \frac{1}{s-3}$$

$$y(s) [s^2 - (0 + s)] = \frac{1 + s(s-3)}{s-3}$$

$$y(s) = \frac{s^2 - s}{(s-3)(s-1)(s-4)} = \frac{A}{s-3} + \frac{B}{s-1} + \frac{C}{s-4}$$

$$A = \frac{s^2 - s}{(s-1)(s-4)} \Big|_{s=3} = \frac{2(3) - 5}{(1-1)(3-4)} = -1$$

$$B = \frac{s^2 - s}{(s-3)(s-4)} \Big|_{s=1} = \frac{2(1) - 5}{(1-3)(1-4)} = -\frac{1}{2}$$

$$C = \frac{s^2 - s}{(s-3)(s-1)} \Big|_{s=4} = \frac{2(4) - 5}{(4-3)(4-1)} = \frac{3}{2}$$

$$y(s) = \frac{-1}{s-3} - \frac{1}{2} \left(\frac{1}{s-1} \right) + \frac{3}{2} \left(\frac{1}{s-4} \right)$$

$$y(t) = -e^{3t} - \frac{1}{2} e^{t} + \frac{3}{2} e^{4t}$$