ASSIGNMENT 5

1. $dy/dt + 3y = e^{-2t}$ at $t = 0,\ y = 2$

$$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$$

$$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$$

$$sy(s) + 3y(s) = \frac{1}{s+2} + 2$$

$$y(s)(s+3) = \frac{1 + 2s + 4}{s+2}$$

$$y(s) = \frac{2s+5}{(s+2)(s+3)}$$

$$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$= \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$$

$$2s + 5 = A(s+3) + B(s+2)$$

At $s = -2$

$$2(-2) + 5 = A(-2+3)$$

$$A = 1$$

At $s = -3$

$$2(-3) + 5 = B(-3+2)$$

$$B = 1$$

$$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$$

$$\therefore y(t) = e^{-2t} + e^{-3t}$$

2. $2\,dy/dt - 6y = \sin 2t$ at $t = 0,\ y = 1$

$$2[sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$$

$$2sy(s) - 2y(0) - 6y(s) = \frac{2}{s^2+4}$$

$$2sy(s) - 2 - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 6y(s) = \frac{2}{s^2+4} + 3$$

$$y(s)(3s-6) = \frac{2 + 3(s^2+4)}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)}$$

$$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{(s^2+4)} + \frac{C}{3s-6} \qquad C = 13/4$$

$$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$$

$$3 = 3B + C$$

$$3B = -1/4$$

$$B = -1/12$$

$$2A - 6B = 1$$

$$3A = 11$$

$$3A = 6(-1/12)$$

$$A = -1/6$$

$$y(s) = -\frac{1}{6}\left(\frac{1}{s^2+4}\right) - \frac{1}{12}\left(\frac{s}{s^2+4}\right) + \frac{13}{4}\left(\frac{1}{3s-6}\right)$$

$$= -\frac{1}{6}\left(\frac{1}{s^2+2^2}\right) - \frac{1}{12}\left(\frac{s}{s^2+2^2}\right) + \frac{13}{12}\left(\frac{1}{s-2}\right)$$

$$\therefore y(t) = -\frac{1}{12}\sin 2t - \frac{1}{12}\cos 2t + \frac{13}{12}e^{2t}$$

3) $dy/dt - 4y = 8$ at $t = 0,\ y = 2$

$$sy(s) - y(0) - 4y(s) = 8/s$$

$$sy(s) - 2 - 4y(s) = 8/s$$

$$sy(s) - 4y(s) = 8/s + 2$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$