

KENSEIMO ANDY CHRISTOPHER
15/ENGG06/042
MECHANICAL ENGINEERING

ASSIGNMENT 5

$$1) \frac{dy}{dt} + 3y = e^{-2t}$$

$$\text{at } t=0 \text{ and } y=2$$

$$y(0) = 2$$

$$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$$

$$y(s)(s+3) - 2 = \frac{1}{s+2}$$

$$y(s)(s+3) = \frac{1}{s+2} + 2$$

$$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$$

$$y(t) = L^{-1}\{y(s)\} = L^{-1}\left[\frac{1+2s+4}{(s+2)(s+3)}\right]$$

$$\frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$s = -2 \Rightarrow 1 + 2(-2) + 4 = A(-2+3)$$

$$1 = A, A = 1$$

$$s = -3 \Rightarrow 1 + 2(-3) + 4 = B(-3+2)$$

$$-1 = -B, B = 1$$

$$y(t) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$$

$$y(t) = e^{-2t} + e^{-3t}$$

$$2) \quad 3 \frac{dy}{dt} - 6y = \sin 2t$$

at $t=0, y=1$

$$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$$

$$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$$

$$y(s)(3s-6) = \frac{2}{s^2+4} + 3$$

$$y(s) = \frac{2 + 3(s^2+4)}{(s^2+4)(3s-6)}$$

$$y(s) = \frac{2 + 3s^2 + 12}{(s^2+4)(3s-6)}$$

$$y(x) = \mathcal{L}^{-1} \left\{ \frac{3s^2 + 14}{(s^2+4)(3s-6)} \right\}$$

$$\frac{3s^2 + 14}{(s^2+4)(3s-6)} = \frac{As + B}{s^2+4} + \frac{C}{3s-6}$$

$$3s^2 + 14 = As + B(3s-6) + C(s^2+4)$$

$$3s^2 + 14 = 3As^2 - 6As + 3Bs - 6B + Cs^2 + 4C$$

$$3A + C = 3 \quad \text{--- (1)} \quad C = 3 - 3A$$

$$3B - 6A = 0 \quad \text{--- (2)} \quad 4(3 - 3A) - 6B = 14$$

$$4C - 6B = 14 \quad \text{--- (3)} \quad 12 - 12A - 6B = 14$$

$$A = -\frac{1}{12}, \quad B = -\frac{1}{6}, \quad C = \frac{13}{4}$$

$$4) \frac{d^2 y}{dt^2} - 2 \frac{dy}{dt} + 5y = e^{2t} \quad \text{at } t=0, y=2, y'=1$$

$$y(0) = 2, \quad y'(0) = 1$$

$$s^2 y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s - 1 + 2 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} - 1 + 2s$$

$$y(s) = \frac{1 - (s-2) + 2s(s-2)}{(s-2)(s^2 - 2s + 5)}$$

$$y(s) = \frac{1 - s + 2 + 2s^2 - 4s}{(s-2)(s^2 - 2s + 5)} = \frac{2s^2 - 5s + 3}{(s-2)(s^2 - 2s + 5)}$$

$$y(t) = L^{-1}\{y(s)\} = L^{-1}\left\{\frac{2s^2 - 5s + 3}{(s-2)(s^2 - 2s + 5)}\right\}$$

$$\frac{2s^2 - 5s + 3}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$$

$$2s^2 - 5s + 3 = As^2 - 2As + 5A + Bs^2 - 2Bs + C - 2C$$

$$2s^2 - 5s + 3 = s^2(A+B) + s(-2A-2B+C) + 5A-2C$$

$$A+B = 2 \quad \text{--- (1)}$$

$$-2A-2B+C = -5 \quad \text{--- (2)}$$

$$5A-2C = 3 \quad \text{--- (3)}$$

$$A = \frac{1}{5}, \quad B = \frac{9}{5}, \quad C = -1$$

$$5) \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t} \quad t=0, y=0, y'=2$$

$$y(0) = 0, y'(0) = 2$$

$$s^2 y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \frac{1}{s-3}$$

$$s^2 y(s) - s(0) - 2 - 6sy(s) + 6(0) + 8y(s) = \frac{1}{s-3}$$

$$y(s)(s^2 - 6s + 8) - 2 = \frac{1}{s-3}$$

$$y(s) = \frac{1 + 2(s-3)}{(s-3)(s^2 - 6s + 8)} = \frac{1 + 2s - 6}{(s-3)(s-4)(s-2)}$$

$$y(t) = L^{-1}\{y(s)\} = L^{-1}\left\{\frac{2s-5}{(s-2)(s-3)(s-4)}\right\}$$

$$\frac{2s-5}{(s-2)(s-3)(s-4)} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s-4}$$

$$s=2 \Rightarrow 2(2) - 5 = A(2-3)(2-4)$$

$$-1 = 2A, \quad A = -\frac{1}{2}$$

$$s=3 \Rightarrow 2(3) - 5 = B(3-2)(3-4)$$

$$1 = -B, \quad B = -1$$

$$s=4 \Rightarrow 2(4) - 5 = C(4-2)(4-3)$$

$$3 = 2C, \quad C = \frac{3}{2}$$

$$y(t) = L^{-1}\left\{-\frac{1}{2} \cdot \frac{1}{(s-2)} - \frac{1}{(s-3)} + \frac{3}{2} \cdot \frac{1}{(s-4)}\right\}$$

$$y(t) = -\frac{1}{2} e^{2t} - e^{3t} + \frac{3}{2} e^{4t}$$

$$y(t) = \frac{1}{2} (3e^{4t} - 2e^{3t} - e^{2t})$$