OLUABUNWA REJOICE NKEMDINACHI
15/ENG03/027
CIVIL ENGINEERING
ASSIGNMENT IV

1) $\frac{dy}{dt} + 3y = e^{-2t}$ given that at $t=0, y=1$

$L\{dy/dt\} = sy(s) - y(0)$

$L\{3y\} = 3y$

$L\{e^{-2t}\} = 1/(s+2)$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s) = \frac{1+2(s+2)}{(s+2)}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

$2s+5 = As + 3A + Bs + 2B$

$A+B = 2 \quad \times 3$

$3A + 2B = 5 \times 1$

$3A + 3B = 6$ — (3)

$3A + 2B = 5$ — (4)

eqn (3) − (4)

$B = 1$

$A + 1 = 2$

$A = 2 - 1$

$A = 1$

$\frac{2s+5}{(s+2)(s+3)} = \frac{1}{s+2} + \frac{1}{s+3}$

$L^{-1}\{y(s)\} = L^{-1}\{\frac{1}{s+2} + \frac{1}{s+3}\}$

$y = e^{-2t} + e^{-3t}$

2) $\frac{3dy}{dt} - 6y = 5\sin 2t$ given that at $t=0, y=1$

$L\{3\,dy/dt\} = 3(sy(s) - y(0))$

$L\{-6y\} = -6y(s)$

$L\{5\sin 2t\} = \frac{2}{s^2+2^2}$

$3sy(s) - 3y(s) - 6y(s) = \frac{1}{s+2}$

$s^2+4s+24$

$3sy(s) - 6y(s) - 3 = \frac{2}{(s+2)^2}$

$y(s)(3s-6) = \frac{2}{(s+2)^2} + 3$

$y(s)(3s-6) = \frac{2+3(s+2)^2}{(s+2)^2}$

$y(s) = \frac{2+3(s+2)^2}{(s+2)^2(3s-6)}$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{A}{(s+2)} + \frac{B}{(s+2)^2} + \frac{C}{(3s-6)}$

$2+3(s+2)^2 = A(s+2)(3s-6) + B(3s-6) + C(s+2)^2$

$2+3s^2+12s+12 = As^2 - A12 + 3Bs - 6B + Cs^2 + 4Cs + 4C$

$3A + C = 3$ ——①

$3B + 4C = 12$ ——②

$-12A - 6B + 4C = 14$ ——③

from ①

$3A = 3 - C$

$A = \frac{3-C}{3}$

$3B + 4C = 12$

$-12\left(\frac{3-C}{3}\right) - 6B + 4C = 14$

$-12 + 4C - 6B + 4C = 14 \times -6$

$-6B + 8C = 28$

$-18B - 24C = -72$

$-18B + 24C = 84$

$48C = -156$

$C = 13/4$

from (2)

$3B = 12 - 13$

$B = -\frac{1}{3}$

from ①

$3A = 3 - C$

$3A = 3 - \frac{13}{4}$

$A = -1/12$

$\frac{2+3(s+2)^2}{(s+2)^2(3s-6)} = \frac{-1/2}{(s+2)} + \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}$

$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{-1/12}{(s+2)} - \frac{1/3}{(s+2)^2} + \frac{13/4}{(3s-6)}\right\}$

$y = -\frac{1}{12}e^{-2t} - \frac{1}{3}te^{-4t} + \frac{13}{12}e^{3t}$

$y = -\frac{1}{12}\left(e^{-2t} + 4te^{-4t} - 13e^{3t}\right)$

3) $\frac{dy}{dt} - 4y = 8$ given that $t=0, y=2$

$L\{\frac{dy}{dt}\} = sy(s) - y(s)$

$L\{-4y\} = -4y(s)$

$L\{8\} = 8/s$

$sy(s) - y(s) - 4y(s) = 8/s$

$sy(s) - 4y(s) - y(s) = 8/s$

$y(s)(s-4) - 2 = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8+2s = A(s-4) + Bs$

$8+2s = As - 4A + Bs$

$A+B = 2$

$-4A = 8$

$A = -2$

$B = 2+2$

$B = 4$

$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$

$L^{-1}\{y(s)\} = L^{-1}\{\frac{-2}{s} + \frac{4}{s-4}\}$

$y = -2 + 4e^{4t}$

(ii) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ given that at $t=0$

$t=0, y=2, y'=1$

$L\{\frac{d^2y}{dt^2}\} = s^2y(s) - sy(s) - y'(s)$

$L\{-2\frac{dy}{dt}\} = -2sy(s) + 2y(s)$

$L\{5y\} = 5y(s)$

$L\{e^{2t}\} = 1/s-2$

$s^2y(s) - sy(s) - y'(s) - 2sy(s) + 2y(s) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s)(s^2-2s+5) = 1/s-2 + 2s - 3$

$y(s)(s^2-2s+5) = \frac{(2s-3)(s-2)}{(s-2)}$

$y(s) = \frac{1+2s^2-5+6}{(s-2)(s^2-2s+5)}$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s-5)}$$

$$\frac{2s^2-7s+7}{(s+2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}$$

$$2s^2 - 7s + 7 = A(s^2-2s+5) + (Bs+C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$A + B = 2 \quad \text{---} (1)$$

$$-2A - 2B + C = -7 \quad \text{---} (2)$$

$$5A - 2C = 7 \quad \text{---} (3)$$

from (1)

$$B = 2 - A$$

from (2)

$$-2A - 2(2-A) + C = -7$$

$$-2A - 4 + 2A + C = -7$$

$$C = -3$$

from (3)

$$5A - 2(-3) = 7$$

$$5A = 7 - 6$$

$$A = 1/5$$

$$A + B = 2$$

$$1/5 + B = 2$$

$$B = 2 - 1/5$$

$$B = 9/5$$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5 s - 3}{(s^2-2s+5)}$$

$$= \frac{1/5}{(s+2)} + \frac{9}{5}\left(\frac{s-1+1}{s^2-2s+5}\right) - \frac{3 \times 2/2}{(s^2-2s+5)}$$

$$\frac{1/5}{(s-2)} + \frac{9}{5}\left(\frac{s+1+1}{(s-1)^2+4}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left\{\frac{s-1}{(s-1)^2+2} + \frac{1+2/2}{(s-1)^2+2^2}\right\} - \frac{3}{2}\left[\frac{2}{(s-1)^2+2^2}\right]\right\}$$

$$= L^{-1}\left\{\frac{1/5}{(s-2)} + \frac{9}{5}\left[\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right]\right.$$

$$\left. - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}$$

$$y = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t \cos 2t + \frac{1}{2}(e^t \sin 2t)\right]$$

$$-3/2\,(e^t \sin 2t)$$

5) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$ given that at $t = 0$, $y = 0$, $y' = 2$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y(s) - sy(s) - y'(0)$$

$$L\left\{-6\frac{dy}{dt}\right\} = -6sy(s) + 6y(s)$$

$$L\{8y\} = 8y(s)$$

$$L\{e^{3t}y\} = \frac{1}{s-3}$$

$$s^2y(s) - sy(s) - y'(0) - 6sy(s) + 6y(s) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 6sy(s) + 8y(s) - 2 = \frac{1}{s-3}$$

$$y(s)[s^2 - 6s + 8) = \frac{1}{s-3} + 2$$

$$y(s)(s^2 - 6s + 8) = \frac{1 + 2(s-3)}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{Bs+C}{(s^2-6s+8)}$$

$$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$$

$$2s-5 = As^2 - 6As + 8A + Bs^2 - 3Bs + Cs - 3C$$

$$A + B = 0$$

$$-6A - 3B + C = 2$$

$$8A - 3C = -5$$

$$B = -A \text{ from } ①$$

$$-6A + 3A + C = 2$$

$$-3A + C = 2 \quad \text{---} ④ \times -3$$

$$8A - 3C = -5 \quad \text{---} ⑤ \times 1$$

$$9A - 3C = -6$$

$$8A - 3C = -5$$

$$A = -1$$

$$B = 1$$

from ④

$$C = 2 - 3$$

$$C = -1$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s^2-6s+8)}$$

$$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s-2)(s-4)}$$

$$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$s-1 = A(s-4) + B(s-2)$$

$$s-1 = As - 4A + Bs - 2B$$

$A+B=1$ —— ① $\times -4$

$-4A-2B=-1$ —— ② $\times 4$

$-4A-4B=-4$

$-4A-2B=-1$

$-2B=-3$

$B=3/2$

$A=-1/2$

$$\frac{s-1}{(s-2)(s-4)} = \frac{-1/2}{s-2} + \frac{3/2}{s-4}$$

$$\frac{2s-5}{(s-3)(s^2-(s+8)} = \frac{-1}{(s-3)} + \left(\frac{-1/2}{(s-2)} + \frac{3/2}{(s-1}\right)$$

$$L^{-1}\{y_s\} = L^{-1}\left\{\frac{-1}{(s-3)} - \frac{1/2}{(s-2)} + \frac{3/2}{(s-4)}\right\}$$

$$y = -e^{3t} - 1/2\, e^{2t} + 3/2\, e^{4t}$$

$$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$$