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DEPT: MECH ENG

COURSE: ENG 381

Assignment V.

1) $\frac{dy}{dt} + 3y = e^{-2t}$

$t=0, y=2$

$sy(s) - y(0) - 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) + 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$L^{-1}\left[\frac{2s+5}{(s+2)(s+3)}\right] = y(s)$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

when $s = -2$

$-4+5 = A(1) + 0 \quad \therefore A = 1$

$A = 1$

when $s = -3$

$-6+5 = A(0) + B(-1)$

$-B = -1$

$B = 1$

$y(s) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$

$y(s) = e^{-2t} + e^{-3t}$

2) $3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0$; $y=1$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6y(s) = \frac{2}{s^2+4}$

$3y(s)(s-2) - 3 = \frac{2}{s^2+4}$

$3y(s)(s-2) = \frac{2}{s^2+4} + 3$

Ruitai paper

$$= \frac{2 + 3s^2 + 12}{s^2+4}$$

$$y(s) = \frac{3s^2+14}{3(s-2)(s^2+4)} = \frac{3s^2+14}{(3s-6)(s^2+4)}$$

$$y(s) = L^{-1}\left[\frac{3s^2+14}{(3s-6)(s^2+4)}\right]$$

$$= L^{-1}\left[\frac{A}{3s-6} + \frac{Bs+C}{s^2+4}\right]$$

$$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$$

at $s=2$

$$12+14 = A(8) + (Bs+C)(0)$$

$$26 = 8A \qquad A = \frac{13}{4}$$

Using the method of coefficients

comparing coefficients

$$3 = A + B \quad \text{—} \quad (i)$$

$$3 = 13/4 + 3B$$

$$3B = 3 - 13/4$$

$$3B = \frac{12-13}{4}$$

$$3B = \frac{-1}{4} \qquad B = \frac{-1}{12}$$

$$14 = 4A - 6C$$

$$14 = \frac{4 \times 13}{4} - 6C$$

$$14 = 13 - 6C$$

$$1 = -6C$$

$$C = -1/6$$

$$y(s) = L^{-1}\left[\frac{13}{4} \times \frac{1}{3s-6} + \frac{-\frac{1}{12}s - \frac{1}{6}}{s^2+4}\right]$$

$$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2} - \frac{1}{12}s \times \frac{1}{s^2+4} - \frac{1}{6} \times \frac{1}{s^2+4}\right]$$

$$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2}\right] + L^{-1}\left[\frac{-1}{12}\frac{s}{s^2+4}\right] + L^{-1}\left[\frac{-1}{6 \times 2} \times \frac{2}{s^2+4}\right]$$

$$= \frac{13}{12}e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t$$

3) $\frac{dy}{dt} - 4y = 8$

at $t=0$, $y=2$

$$sy(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$sy(s) - 2 - 4y(s) = 8/s$$

$$y_s(s-4) = \frac{8}{s} + 2$$

$$y_s(s-4) = \frac{8-2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$y(s) = L^{-1}\left[\frac{8+2s}{s(s-4)}\right] = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$$

$$\frac{8+2s}{s(s-4)} = \frac{A(s-4)+B(s)}{s(s-4)}$$

$8+2s = A(s-4) + B(s)$

when $s=4$

$8+8 = A(0) + 4B$

$16 = 4B$

$B = 4$

when $s = 0$

$8 = -4A + 0$

$-4A = 8$

$A = -2$

$$y = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$$

$y = -2 + 4e^{4t}$

4) $$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

at $t=0$, $y=2$, $y'=1$

$s^2y(s) - s(2) - 1 - 2(sy(s) - 2) + 5y(s)$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s)$

$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$$y(s) = \frac{1 - 2s(s-2) - 3(s-2)}{s-2} \cdot \frac{1}{s^2-2s+5}$$

$$y(s) = \frac{1 - 2s^2 + 4s - 3s + 6}{s-2} \cdot \frac{1}{s^2-2s+5}$$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2-2s+5)}$$

$$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A+Bs}{(s^2-2s+5)} + \frac{C}{s-2}$$

$2s^2 - 7s + 7 = A + Bs(s-2) + C(s^2 - 2s + 5)$

$2s^2 - 7s + 7 = As + Bs^2 - 2A - 2Bs + Cs^2 - 2Cs + 5C$

Comparing coefficients

s^2: $B + C = 2 - 1$ $B = 2 - C$

s: $A - 2B - 2C = -7 - 2$

c: $-2A + 5C = 7 \rightarrow 3$

$A - 2(2 - C) - 2C = -7$

$A - 4 + 2C - 2C = -7$

$A - 4 = -7$

$A = -3$

$-2(-3) + 5C = 7$

$5c = 7-6$

$5c = 1$

$c = 1/5$

$B = 2 - \frac{1}{5} = \frac{9}{5}$

$$y(s) = \frac{-3}{s^2-2s+5} + \frac{9/5 s}{s^2-2s+5} + \frac{1/5}{s-2}$$

$$y(s) = \frac{-3}{(s-1)^2+4} + \frac{9/5 s}{(s-1)^2+4} + \frac{1/5}{(s-2)}$$

$$y(s) = \frac{-3}{2}\left[\frac{2}{(s-1)^2+4}\right] + \frac{9}{5}\left(\frac{(s-1)}{(s-1)^2+4}\right.$$

$$\left.+ \frac{1 \times 2/2}{(s-1)^2+4}\right] + \left.\frac{1/5}{s-2}\right]$$

$$y = \frac{-3}{2} e^t \sin 2t + \frac{9}{5}\left[e^t \cos 2t\right.$$

$$\left.+ \frac{1}{2} e^t \sin 2t\right] + \frac{1}{5} e^{2t}.$$

5) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$t=0,\ y=0,\ y'=2$

$s^2 y(s) - sy(0) - y'(0) - 6y($

$sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}$

$s y(s) - 2 - 6sy(s) + 8y(s)$

$= 1/s-3$

$y(s) = (s^2-6s+8) - 2 = \frac{1}{s-3}$

$$y(s) = \frac{1}{s-3} + 2 \cdot \frac{1}{(s^2-6s+8)}$$

$$y(s) = \frac{1+2(s-3)}{s-3} \cdot \frac{1}{(s-4)(s-2)}$$

$$y(s) = \frac{2s-4}{(s-3)(s-4)(s-2)} = \frac{A}{(s-3)} + \frac{B}{(s-4)} + \frac{C}{(s-2)}$$

$2s-5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$

at $s = 4$

$8-5 = B(4-3)(4-2)$

$3 = B2$

$B = 3/2$

at $s=2$

$4-5 = C(2-3)(2-4)$

$-1 = 2C$

$C = -1/2$

at $s=3$

$6-5 = A(3-4)(3-2)$

$1 = -A \quad A = -1$

$$\therefore y(s) = \frac{-1}{s-3} + \frac{3/2}{s-4} - \frac{1/2}{s-2}$$

$$y = -e^{3t} + \frac{3}{2} e^{4t} - \frac{1}{2} e^{2t}.$$