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ASSIGNMENT 5

1) $\frac{dy}{dt} + 3y = e^{-2t}$, given that at $t=0$, $y=2$

$\Rightarrow y' + 3y = e^{-2t}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 + 3y(s) = \frac{1}{s+2}$

$sy(s) + 3y(s) = 2 + \frac{1}{s+2}$

$y(s)(s+3) = \frac{2(s+2)+1}{s+2}$

$y(s)(s+3) = \frac{2s+4+1}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3)+B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

when $s=-2$

$2(-2)+5 = A(-2+3)$

$-4+5 = A(1)$

$A = 1$

when $s = -3$

$2(-3)+5 = B(-3+2)$

$-6+5 = B(-1)$

$-1 = -B$

$B = 1$

$\therefore y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = L^{-1}\left\{\frac{1}{s+2}\right\} + L^{-1}\left\{\frac{1}{s+3}\right\}$

$y(t) = e^{-2t} + e^{-3t}$

11) $3\frac{dy}{dt} - 6y = \sin 2t$ given that at $t = 0$, $y = 1$

$\Rightarrow 3y' - 6y = \sin 2t$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3(sy(s) - 1) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3 - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 6y(s) = 3 + \frac{2}{s^2+4}$

$y(s)(3s-6) = \frac{3(s^2+4)+2}{s^2+4}$

$y(s)(3s-6) = \frac{3s^2+12+2}{s^2+4}$

$y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{Bs+C}{s^2+4}$

$\frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A(s^2+4) + (Bs+C)(3s-6)}{(3s-6)(s^2+4)}$

$3s^2 + 14 = A(s^2+4) + (Bs+C)(3s-6)$

when $s = 2$

$3(2)^2 + 14 = A[(2)^2 + 4]$

$12 + 14 = A(4+4)$

$26 = 8A$

$A = \frac{26}{8} = \frac{13}{4}$

Recall:

$3s^2 + 14 = A(s^2+4) + (Bs+C)(3s-6)$

$3s^2 + 14 = As^2 + 4A + 3Bs^2 - 6Bs + 3Cs - 6C$

Comparing coefficients

Coefficients of s^2

$3 = A + 3B$

Sub $A = 13/4$

$3 = 13/4 + 3B$

$$3B = 3 - \frac{13}{4} = \frac{12-13}{4}$$

$$3B = \frac{-1}{4}$$

$$B = \frac{-1}{4} \times \frac{1}{3} = \frac{-1}{12}$$

Coefficients of s

$0 = -6B + 3C$

sub $B = -1/12$

$$0 = -6\left(\frac{-1}{12}\right) + 3C$$

$$0 = \frac{1}{2} + 3C$$

$$3C = \frac{-1}{2}$$

$$C = \frac{-1}{2} \times \frac{1}{3} = \frac{-1}{6}$$

$$y(s) = \frac{13/4}{3s-6} - \frac{1/12 s - 1/6}{s^2+4}$$

$$y(s) = \frac{13/4}{3s-6} - \frac{1/12 s}{s^2+4} - \frac{1/6}{s^2+4} \Rightarrow \frac{13/4}{3(s-2)} \frac{-1/12 s}{s^2+4} \frac{-1/6}{s^2+4}$$

$$y(s) = \frac{13}{4} \times \frac{1}{3} \times \frac{1}{s-2} \frac{-1}{12} \times \frac{s}{s^2+4} \frac{-1}{6} \times \frac{2}{s^2+4} \times \frac{1}{2}$$

$$y(s) = \frac{13}{12}\left(\frac{1}{s-2}\right) \frac{-1}{12}\left(\frac{s}{s^2+4}\right) \frac{-1}{12}\left(\frac{2}{s^2+4}\right)$$

$$y(t) = \frac{13}{12} L^{-1}\left\{\frac{1}{s-2}\right\} \frac{-1}{12} L^{-1}\left\{\frac{s}{s^2+4}\right\} \frac{-1}{12} L^{-1}\left\{\frac{2}{s^2+4}\right\}$$

$$y(t) = \frac{13}{12} e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t$$

iii) $\frac{dy}{dt} - 4y = 8$ given that at $t = 0$, $y = 2$

$\Rightarrow y' - 4y = 8$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$sy(s) - 4y(s) = 2 + \frac{8}{s}$

$y(s)(s-4) = \frac{2s+8}{s}$

$y(s) = \frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$\frac{2s+8}{s(s-4)} = \frac{A(s-4) + Bs}{s(s-4)}$

$2s+8 = A(s-4) + Bs$

when $s = 0$

$2(0) + 8 = A(0-4)$

$8 = -4A$

$A = -2$

when $s = 4$

$2(4) + 8 = B(4)$

$8 + 8 = 4B$

$16 = 4B$

$B = 4$

$\therefore y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -L^{-1}\left\{\frac{2}{s}\right\} + 4L^{-1}\left\{\frac{1}{s-4}\right\}$

$y(t) = -2 + 4e^{4t}$

$y(t) = 4e^{4t} - 2$

iv) $\frac{d^2y}{dt^3} - \frac{2dy}{dt} + 5y = e^{2t}$ given that at $t=0$, $y=2$, $y'=1$

$\Rightarrow y'' - 2y' + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

sub $y(0) = 2$ & $y'(0) = 1$

$s^2y(s) - 2s - 1 - 2(sy(s) - 2) + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = 2s - 3 + \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s(s-2) - 3(s-2) + 1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 - 4s - 3s + 6 + 1}{s-2}$

$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s + 5}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A(s^2 - 2s + 5) + (Bs + C)(s-2)}{(s-2)(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

when $s = 2$

$2(2)^2 - 7(2) + 7 = A[(2)^2 - 2(2) + 5]$

$8 - 14 + 7 = A(4 - 4 + 5)$

$1 = 5A$

$A = 1/5$

Recall:

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5s + Bs^2 - 2Bs + Cs - 2C$

Comparing coefficients

Coefficients of s^2

$2 = A + B$

Sub $A = 1/5$

$2 = 1/5 + B$

$B = 2 - \frac{1}{5} = \frac{10-1}{5}$

$B = 9/5$

Coefficients of s

$-7 = -2A - 2B + C$

Sub $A = 1/5$ & $B = 9/5$

$-7 = \frac{-2}{5} - \frac{18}{5} + C$

$C = \frac{2}{5} + \frac{18}{5} - 7$

$C = \frac{2 + 18 - 35}{5} = \frac{-15}{5}$

$C = -3$

$$\therefore y(s) = \frac{1/5}{s-2} + \frac{9/5\, s - 3}{s^2 - 2s + 5}$$

$$y(s) = \frac{1/5}{s-2} + \frac{9/5\, s - 3}{(s-1)^2 + 4}$$

$$y(s) = \frac{1/5}{s-2} + \frac{9/5\, s - 9/5 - 6/5}{(s-1)^2 + 4}$$

$$y(s) = \frac{1/5}{s-2} + \frac{9/5(s-1) - 6/5}{(s-1)^2 + 4}$$

$$y(s) = \frac{1/5}{s-2} + \frac{9/5(s-1)}{(s-1)^2 + 4} - \frac{6/5}{(s-1)^2 + 4}$$

$$y(s) = \frac{1/5}{s-2} + \frac{9}{5}\left(\frac{s-1}{(s-1)^2 + 4}\right) - \frac{6}{5} \times \frac{1}{2} \times \frac{2}{(s-1)^2 + 4}$$

$$y(s) = \frac{1/5}{s-2} + \frac{9}{5}\left(\frac{s-1}{(s-1)^2 + 4}\right) - \frac{3}{5}\left(\frac{2}{(s-1)^2 + 4}\right)$$

$$y(t) = \frac{1}{5} L^{-1}\left\{\frac{1}{s-2}\right\} + \frac{9}{5} L^{-1}\left\{\frac{s-1}{(s-1)^2 + 4}\right\} - \frac{3}{5} L^{-1}\left\{\frac{2}{(s-1)^2 + 4}\right\}$$

$$y(t) = 1/5\, e^{2t} + 9/5\, e^{t}\cos 2t - 3/5\, e^{t}\sin 2t$$

v) $\dfrac{d^2y}{dt^2} - \dfrac{6dy}{dt} + 8y = e^{3t}$ given that at $t = 0$, $y = 0$, $y' = 2$

$\Rightarrow y'' - 6y' + 8y = e^{3t}$

$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s) = \dfrac{1}{s-3}$

sub $y(0) = 0$ & $y'(0) = 2$

$s^2y(s) - 2 - 6sy(s) + 8y(s) = \dfrac{1}{s-3}$

$s^2y(s) - 6sy(s) + 8y(s) = 2 + \dfrac{1}{s-3}$

$y(s)(s^2 - 6s + 8) = \dfrac{2(s-3) + 1}{s-3}$

$y(s) = \dfrac{2s - 6 + 1}{(s^2 - 6s + 8)(s-3)}$

$y(s) = \dfrac{2s - 5}{(s-4)(s-2)(s-3)} = \dfrac{A}{s-4} + \dfrac{B}{s-2} + \dfrac{C}{s-3}$

$\dfrac{2s-5}{(s-4)(s-2)(s-3)} = \dfrac{A(s-2)(s-3) + B(s-4)(s-3) + C(s-4)(s-2)}{(s-4)(s-2)(s-3)}$

$2s - 5 = A(s-2)(s-3) + B(s-4)(s-3) + C(s-4)(s-2)$

when $s = 4$	when $s = 2$	when $s = 3$
$2(4) - 5 = A(4-2)(4-3)$	$2(2) - 5 = B(2-4)(2-3)$	$2(3) - 5 = C(3-4)(3-2)$
$8 - 5 = A(2)(1)$	$4 - 5 = B(-2)(-1)$	$6 - 5 = C(-1)(1)$
$3 = 2A$	$-1 = 2B$	$1 = -C$
$A = 3/2$	$B = -1/2$	$C = -1$

$\therefore y(s) = \dfrac{3/2}{s-4} - \dfrac{1/2}{s-2} - \dfrac{1}{s-3}$

$y(t) = \dfrac{3}{2} L^{-1}\left\{\dfrac{1}{s-4}\right\} - \dfrac{1}{2} L^{-1}\left\{\dfrac{1}{s-2}\right\} - L^{-1}\left\{\dfrac{1}{s-3}\right\}$

$y(t) = 3/2\, e^{4t} - 1/2\, e^{2t} - e^{3t}$