

... ..  
 ... ..  
 ... ..

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$\begin{bmatrix} 1 & 1 \\ s+2 & s+3 \end{bmatrix} = e^{-2t} + e^{-3t}$$

$$y'' + y' + y = 1$$

$$y = 0, y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$y'' + y' + y = 1$$

$$Y(s) = \frac{3s^2 + 14s + 11}{(s+2)(s-2)(s+4)}$$

$$= \frac{A}{s+2} + \frac{B}{s-2} + \frac{C}{s+4}$$

$$3s^2 + 14s + 11 = A(s-2)(s+4) + B(s+2)(s+4) + C(s+2)(s-2)$$

Comparing coefficients

$$3 = A + B + C$$

$$14 = -4A + 6B + 4C$$

$$11 = 4A - 2B - 8C$$

$$L \left[ \frac{13/4}{3s-6} + \frac{(-1/2)s - 1/2}{s^2 + 4} \right]$$

$$= \left[ \frac{13}{12} e^{3t/2} - \frac{1}{2} \cos 2t - \frac{1}{2} \sin 2t \right]$$

iii)  $\frac{dy}{dt} - 4y = 8$

given that  $t=0, y=2$

$$-sY(s) - 4Y(s) = 8/s$$

$$sY(s) - 2 - 4Y(s) = 8/s$$

$$sY(s) - 4Y(s) = 8/s + 2$$

$$sY(s) - 4Y(s) = \frac{8+2s}{s}$$

$$Y(s) [s-4] = \frac{8+2s}{s}$$

$$Y(s) = \frac{s+2s}{s(s-4)}$$

$$\frac{A}{s} + \frac{B}{s-4} = \frac{8+2s}{s(s-4)}$$

$$A(s-4) + 4B = 8+2s$$

$$\text{let } s=0$$

$$-4A = 8$$

$$A = -2$$

$$\text{let } s=4$$

$$4B = 8+8$$

$$B = 4$$

$$L \left[ \frac{-2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

$$v) \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$t=0, y=2, y'=1$$

$$s^2Y(s) - sY(s) - y'(0) - 2(sY(s) - Y(0)) + 5Y(s) = \frac{1}{s-2}$$

$$s^2Y(s) - 2s - 1 - 2[sY(s) - 2] + 5Y(s) = \frac{1}{s-2}$$

$$s^2Y(s) - 2s + 3 - 2sY(s) + 5Y(s) = \frac{1}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 3}{(s-2)(s^2 - 2s + 5)}$$

$$Y(s) = \frac{2s^2 - 7s + 3}{(s-2)(s^2 - 2s + 5)}$$

$$Y(s) = \frac{2s^2 - 7s + 3}{(s-2)(s^2 - 2s + 5)}$$

$$\frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5} = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)}$$

$$A(s^2-2s+5) + Bs+C(s-2) = 2s^2-7s+7$$

Let  $s=2$

$$A(5) = 1$$

$$A = 1/5$$

Comparing coefficients

$$A+B = 2$$

$$B = 2 - 1/5$$

$$B = 9/5$$

$$5A + 2C = 7$$

$$1 + 2C = 7$$

$$C = 3$$

$$L \left[ \frac{1/5}{s-2} + \frac{(-9/5)s+3}{s^2-2s+5} \right] = L \left[ \frac{1/5}{s-2} + \frac{(-9/5s^2)}{s^2-2s+5} + \frac{3}{s^2-2s+5} \right]$$

$$L \left[ \frac{1/5 \cdot 1/s}{s-2} + \frac{3 \cdot 3/2}{(s-2)^2 + 2^2} \right]$$

$$L \left[ \frac{1/5 \cdot 1/s}{s-2} + \frac{2 \cdot 3/2}{(s-2)^2 + 2} \right]$$

$$= 1/5 e^{2t} \cdot \frac{9}{5} e^{2t} \cos 2t + 3/2 e^{2t} \sin 2t$$