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15/ENG06/030

Mechanical Engineering

300 Level

1 $\frac{d^2y}{d\theta^2} + 4\frac{dy}{d\theta} + 5y = 6\sin\theta$

Solution

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-4 \pm \sqrt{4^2 - 4\times1\times5}}{2\times1}$

$= \frac{-4 \pm \sqrt{-4}}{2}$

$m = -2 \pm j$

$y = e^{-2\theta}[C\cos\theta + D\sin\theta$

Partial Integral

$y = A\cos\theta + B\sin\theta$

$\frac{dy}{d\theta} = -A\sin\theta + B\cos\theta$

$\frac{d^2y}{d\theta^2} = -A\cos\theta - B\sin\theta$

$\therefore [-A\cos\theta - B\sin\theta] + 4[-A\sin\theta + B\cos\theta] + 5[A\cos\theta + B\sin\theta] = 6\sin\theta$

$-A\cos\theta - B\sin\theta - 4A\sin\theta + 4B\cos\theta + 5A\cos\theta + 5B\sin\theta = 6\sin\theta$

$-A\cos\theta + 4B\cos\theta + 5A\cos\theta - B\sin\theta - 4A\sin\theta + 5B\sin\theta = 6\sin\theta$

Comparing coefficients

$-A + 4B + 5A = 0$

$4A + 4B = 0$ —— (i)

$-B - 4A + 5B = 6$

$-4A + 4B = 6$ —— (ii)

adding eqn (1) & (2)

$8B = 6$

$B = \frac{6}{8} = \frac{3}{4}$

$4A + 4\left(\frac{3}{4}\right) = 0$

$4A = -3$

$A = \frac{-3}{4}$

$y = -\frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$ —— (iy)

$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$

iii) at $\theta = \infty$

$\frac{dy}{d\theta} = e^{-2\theta}[-C\sin\theta + D\cos\theta] + [C\cos\theta + D\sin\theta] - 2e^{-2\theta} + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

at $\theta = \infty$ and $\frac{dy}{d\theta} = 0$

$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$

$-\frac{3}{4}\sin\theta = \frac{3}{4}\cos\theta$

$-\cos\theta = \sin\theta$

Divide both sides by $-\cos\theta$

$1 = \frac{\sin\theta}{-\cos\theta}$

$-\tan\theta = 1$

$\theta = \tan^{-1}(-1)$

$\theta = -45^{\circ}$

2 $EI\frac{d^3y}{dx^2} = \frac{w}{2}(L-x)^2$

Solution

$EI\, m^2 = 0$

$m^2 = 0$

$m = \pm\sqrt{0}$

$m = \pm 0$

$$4A + 4\left(\frac{3}{4}\right) = 0$$

$$4A = -3$$

$$A = \frac{-3}{4}$$

$$y = \frac{-3}{4}\cos\theta + \frac{3}{4}\sin\theta \quad \text{(iiy)}$$

$$y = e^{-2\theta}(C\cos\theta + D\sin\theta) - \frac{3}{4}\cos\theta + \frac{3}{4}\sin\theta$$

iii) at $\theta = \infty$

$$\frac{dy}{d\theta} = e^{-2\theta}[-C\sin\theta + D\cos\theta] + [C\cos\theta + D\sin\theta] - 2e^{-2\theta} + \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

at $\theta = \infty$ and $\frac{dy}{d\theta} = 0$

$$0 = \frac{3}{4}\sin\theta + \frac{3}{4}\cos\theta$$

$$-\frac{3}{4}\sin\theta = \frac{3}{4}\cos\theta$$

$$-\cos\theta = \sin\theta$$

Divide both sides by $-\cos\theta$

$$1 = \frac{\sin\theta}{-\cos\theta}$$

$$-\tan\theta = 1$$

$$\theta = \tan^{-1}(-1)$$

$$\theta = -45^\circ$$

2. $$EI\frac{d^2y}{dx^2} = \frac{w}{2}(L-x)^2$$

Solution

$$EI\,m^2 = 0$$

$$m^2 = 0$$

$$m = \pm\sqrt{0}$$

$$m = \pm 0$$

$y = e^{0x}(A+Bx)$

$y = A+Bx$ ——— CF (1)

To obtain particular Integral

$y = Px^2 + Qx^3 + Rx^4$

$\frac{dy}{dx} = 2Px + 3Qx^2 + 4Rx^3$

$\frac{d^2y}{dx^2} = 2P + 6Qx + 12Rx^2$

$EI[2P + 6Qx + 12Rx^2] = \frac{w}{2}(L-x)^2$

$2PEI + 6QEIx + 12Rx^2EI = \frac{w}{2}(L^2 - 2Lx + x^2)$

$4PEI + 12QEIx + 24REIx^2 = w(L^2 - 2Lx + x^2)$

$24REI = w$

$R = \frac{w}{24EI}$ ——— (2)

$12QEI = -2wL$

$Q = \frac{-2wL}{12EI} = \frac{-wL}{6EI}$ ——— (3)

$4PEI = wL^2$

$P = \frac{wL^2}{4EI}$

$y = \left[\frac{wL^2}{4EI}\right]x^2 - \left[\frac{wL}{6EI}\right]x^3 + \left[\frac{w}{24EI}\right]x^4$

$= \frac{wL^2x^2}{4EI} - \frac{wLx^3}{6EI} + \frac{wx^4}{24EI}$

$= \frac{6wL^2x^2 - 4wLx^3 + wx^4}{24EI}$ ——— (4) (PI)

General solution becomes.

$$y = A + Bx + \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

at $y = 0$, $x = 0$, $\frac{d^2y}{dx^2} = 0$

$$0 = A + B(0) + \frac{w}{24EI}\left[6L^2(0) - 4L(0)^3 + 0\right]$$

$A = 0$

$$\frac{dy}{dx} = B + \frac{w}{24EI}\left[12L^2x - 12Lx^2 + 4x^3\right]$$

$$0 = B + \frac{w}{24EI}\left[4(0) - 12(0) + 4(0)\right]$$

$B = 0$

$$y = \frac{w}{24EI}\left[6L^2x^2 - 4Lx^3 + x^4\right]$$

$$y = \frac{wx^2}{24EI}\left[6L^2 - 4Lx + x^2\right]$$

$$y = \frac{wx^3}{24EI}\left[x^2 - 4Lx + 6L^2\right]$$

when $x = L$

$$y = \frac{wL^2}{24EI}\left[L^2 - 4Lx + 6L^2\right] = \frac{wL^2}{24EI}\left[3L^2\right]$$

$$y = \frac{wL^4}{8EI}$$