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15/ENG07/002
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i) $\frac{dy}{dt} + 3y = e^{-2t}$

$y^{(1)} + 3y \cdot e^{-2t}$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$sy(s) - 2 \quad 3y(s) = \frac{1}{s+2}$

$y(s)\,(s+3) = \frac{1}{s+2} + \frac{2}{1} \cdot \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+2}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = \left.\frac{1+2s+4}{s+3}\right|_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

$B = \left.\frac{1+2s+4}{s+2}\right|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

$3\frac{dy}{dt} - 6y = \sin 2t$

$3y^{(1)} - 6y = \sin 2t$

$3[sy(s) - y(s)] - 6y(s) = \frac{s}{s^2+4}$

$3sy(s) - 3y(s) - 6y(s) = \frac{2}{s^2+4}$

$sy(s)\,[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$

$Y(s)\left[3s-6\right] = \dfrac{2+3s^2+12}{(s^2+4)} = \dfrac{3s^2+14}{(s^2+4)}$

$Y(s) = \dfrac{3s^2+14}{(s^2+4)(3s-6)} = \dfrac{A+(B)s}{(s^2+4)} + \dfrac{C}{3s-6}$

$C: \left.\dfrac{3s^2+14}{s^2+14}\right|_{s=2} = \dfrac{3(2)^2+14}{2^2+4} = \dfrac{13}{14}$

$3s^2+14 = (A+Bs)(3s-6)+C(s^2+4)$

$3s^2+14 = 3As-6A+3Bs^2-6Bs+Cs^2+4C$

$3 = 3B+C$ where $C = 13/14$]

$3 \; 3B+\dfrac{B}{4}$ $3B = -1/4$, $B = \dfrac{-1}{12}$

$3A-6B=0$

$3A = 6B$

$3A = 6\times\dfrac{-1}{12}$ $A = \dfrac{-1}{6}$

$Y(s) = \dfrac{-1/6-(-1/12)s}{s^2+4} + \dfrac{13/14}{3s-6}$

$= \dfrac{-1/6}{s^2+4} - \dfrac{-1/12\, s}{s^2+4} + \dfrac{13/14}{3s-6}$

$= \dfrac{-1}{6}\cdot\dfrac{1}{s^2+2^2} - \dfrac{1}{12}\dfrac{s}{s^2+2^2} + \dfrac{13}{4}\dfrac{1}{3(s-2)}$

$= \dfrac{-1}{6}\cdot\dfrac{1}{2}\left[\dfrac{2}{s^2+2^2}\right] - \dfrac{1}{12}\left[\dfrac{s}{s^2+2^2}\right] + \dfrac{13}{2}\left[\dfrac{1}{s-2}\right]$

$y(t) = \dfrac{-1}{12}\sin 2t - \dfrac{1}{12}\cos 2t + \dfrac{13}{2}e^{2t}$

$y(t) = \dfrac{1}{12}\left[-\sin 2t - \cos 2t + 13e^{2t}\right]$

$y(t) = \dfrac{1}{12}\left[13e^{2t} - (\cos 2t - \sin 2t)\right]$

3) $\frac{\partial y}{\partial t} - 4y = 8 \qquad t=0 \quad y=2 \qquad y_{(0)} = 2$

$y^{(1)} - 4y = 8$

$sy(s) - y(0) - 4y(s) = 8/s$

$sy(s) - 2 - 4y(s) = 8/s$

$y(s)(s-4) = 8/s + \frac{2}{1} \quad \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$A = \left.\frac{8+2s}{s-4}\right|_{s=0} = \frac{8}{-4} = -2$

$B = \left.\frac{8+2s}{s}\right|_{s=4} = \frac{8+2(4)}{4} = 4$

$y(s) = \frac{-2}{s} + \frac{4}{s-4}$

$y(t) = -2 + 4e^{4t}$

4] $\frac{\partial^2 y}{\partial x^2} - \frac{2\partial y}{\partial x} + 5y = e^{2t}$ at $t=0$ ; $y=2$ , $y^{(1)} = 1$

$y(0) = 2 \quad y^{(1)}(0) = 1$

$y(s) - 2y(s) + 5y = e^{2t}$

$(s^2 y(s) - sy(0) - y(0)] - 2(sy(s) - y(0)) + 5y(s)$

$= \frac{1}{s-2}$

$s^2 y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$y(s)[s^2 - 2s + 5] = 1/s-2 + \frac{2s}{1} - \frac{3}{1} = \frac{1+2s(s-2)-3(s-2)}{s-2}$

$y(s)[s^2 - 2s + 5) = \frac{1+2s^2-4s-3s+6}{s-2} = \frac{2s^2-7s+7}{s-2}$

$$y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{s-2} + \frac{Bs + C}{s^2 - 2s - 5}$$

$$A = \frac{2s^2 - 7s + 7}{s^2 - 2s + 5}\Bigg|_{s=2} = \frac{2(2)^2 - 7(2) + 7}{2^2 - 2(2) + 5} = \frac{1}{5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + C)(s-2)$$

$$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$$

$$2 = A + B$$

$$2 = \frac{1}{5} + B = \cdot 9/5$$

$$7 = 5A - 2C$$

$$7 = 5(1/5) - 2C$$

$$C = -2$$

$$y(s) = \frac{1/5}{s-2} + \frac{\frac{9}{5}s - 3}{s^2 + 2s + 5} = \frac{1}{5}\frac{1}{s-2} + \frac{9/5\ s}{(s+1)^2 + 4} - \frac{3}{(s+1)^2 + 4}$$

$$y(s) = \frac{1}{5}\cdot\frac{1}{s-2} + \frac{9}{5}\frac{s - 1 + 1}{(s+1)^2 + 4} - \frac{3}{(s+1)^2 + 2^2}$$

$$y(s) = \frac{1}{5}\frac{1}{s-2} + \frac{9}{5}\frac{s+1}{(s+1)^2 + 2^2} - \frac{4}{(s+1)^2 + 2^2} : \frac{2}{2}$$

$$y(s) = \frac{1}{5}\cdot\frac{1}{s-2)} + \frac{9}{5}\cdot\frac{s+1}{(s+1)^2 + 2^2} - 2\cdot\frac{2}{(s+1)^2 + 2^2}$$

$$y(t) = 1/5\ e^{2t} + 9/5\ e^{-t}\cos 2t - 2e^{-t}\sin 2t$$

$$\cancel{y(t) = \frac{1}{5} e^{2t} + 9/5\ e^{t}}$$

$$y_t = \frac{1}{5}\left[e^{2t} + 9e^{-t}\cos 2t - 10e^{-t}\sin 2t\right]$$

$$= \frac{1}{5}\left[e^{2t} + e^{-t}(9\cos 2t - 10\sin 2t)\right]$$

8) $\frac{d^2y}{dt^2} - \frac{6dy}{dt} + 8y = e^{3t}$ at $t=0,\ y=0\ \ y'=2$

$$y^{(2)} - 6y^{(1)} + 8y = e^{3t}$$

$$s^2y(s) - sy(0) - y'(0) - 6\,[sy(s) - y(0)] + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = 1/s-3$$

$$y_{(s)}[s^2 - 6s + 8] = \frac{1}{s-3} + \frac{2}{1} = \frac{1 + 2(s-3)}{s-3} = \frac{1+2s-6}{s-3}$$

$$y_{(s)} = \frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{2s-5}{(s-3)(s-2)(s-4)} =$$

$$\frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$A = \frac{2s-5}{(s-2)(s-4)}\Big|_{s=3} = \frac{2(3)-5}{(3-2)(3-4)} = -1$$

$$B = \frac{2s-5}{(s-3)(s-4)}\Big|_{s=2} = \frac{2(2)-5}{(2-3)(2-4)} = \frac{-1}{2}$$

$$C = \frac{2s-5}{(s-3)(s-2)}\Big|_{s=4} \quad \frac{2(4)-5}{(4-3)(4-2)} = \frac{3}{2}$$

$$y_{(s)} = \frac{-1}{s-3} - \frac{1}{2}\cdot\frac{1}{s-2} + \frac{3}{2}\cdot\frac{1}{s-4}$$

$$y_{(t)} = -e^{3t} - 1/2\,e^{2t} + 3/2\,e^{4t}$$

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