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15/ENG061030

Mechanical Engineering

Eng 6381 - Engineering Maths III

1) If  $y = e^{2^2+x}$

Show that

$$y'' = y'(2x+1) + 2y$$

$$y = e^{x^2+x}$$

$$\frac{dy}{dx} = y'$$

$$\text{Let } y = e^u, \quad \frac{dy}{dx} = y' = \frac{dy}{du} \times \frac{du}{dx}$$

$$u = x^2+x$$

$$\frac{dy}{du} = y' = e^u \times 2x+1$$

$$y' = (2x+1)e^u$$

$$y' = (2x+1)e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = y''$$

$$y' = (2x+1)e^{x^2+x}$$

Using product rule

$$\text{Let } u = 2x+1 \quad v = e^{x^2+x}$$

$$\frac{du}{dx} = 2$$

$$\frac{dv}{dx} = 2x+1 e^{x^2+x}$$

$$\frac{d^2y}{dx^2} = y'' = v \frac{du}{dx} + u \frac{dv}{dx}$$

$$= 2(e^{x^2+x}) + 2x+1(2x+1)e^{x^2+x}$$

$$\text{Since } y = e^{x^2+x}$$

$$y' = (2x+1)e^{x^2+x}$$

$$y'' = 2(y) + 2(x+1)(y')$$

$$y' = 2y + (2x+1)y'$$

$$y'' = y'(2x+1) + 2y$$

Hence proving that

$$y^{(n+2)} = (2x+1)y^{(n+1)} + 2(n+1)y^n$$

$$y'' = y'(2x+1) + 2y$$

$$y'' - y'(2x+1) - 2y = 0$$

Using Leibnitz theorem

①  $\cdot y''$

$$u = y'' \quad v = 1$$

$$u^n = y^{n+2} \quad v' = 0$$

$$= u^n v + n u^{n-1} v'$$

$$= y^{n+2} (1) + n (y^{n+1}) (1/n)$$

$$= y^{n+2}$$

②  $-y'(2x+1)$

$$u = y'$$

$$v = -(2x+1)$$

$$u^n = y^{n+1}$$

$$v' = -2$$

$$v'' = 0$$

$$= u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v''$$

$$= y^{n+1} (2x+1) + n (y^n) (-2)$$

$$= -(2x+1) y^{n+1} - 2n y^n$$

③  $-2y$

$$u = y$$

$$v = 2$$

$$u^n = y^n$$

$$v' = 0$$

$$= u^n v + n u^{n-1} v'$$

$$= 2y^n$$

$$y^{n+2} = -(2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2ny^n + 2y^n$$

$$y^{n+2} = (2x+1)y^{n+1} + 2y^n(x+1)$$

$$y^{n+2} = (2x+1)y^{n+1} + 2 \cdot (n+1)y^n$$

2) Using Leibnitz theorem give that

$$y = x^3 e^{4x}$$

determine  $y^{(5)}$

$$v = x^3$$

$$v' = 3x^2$$

$$v'' = 6x$$

$$v''' = 6$$

$$v^{(4)} = 0$$

$$v^{(5)} = 0$$

$$u = e^{4x}$$

$$u' = 4e^{4x}$$

$$u'' = 16e^{4x}$$

$$u''' = 64e^{4x}$$

$$u^{(4)} = 256e^{4x}$$

$$u^{(5)} = 1024e^{4x}$$

$$y^5 = u^5 v + n u^4 v' + \frac{n(n-1)}{2!} u^3 v'' + n(n-1)(n-2) u^2 v''' + \frac{n(n-1)(n-2)(n-3)}{4!} u^2 v^{(4)} + \frac{n(n-1)(n-2)(n-3)(n-4)}{3!} u v^{(5)}$$

$$y^5 = [1024 e^{4x} (x^3)] + [5(256 e^{4x}) 6x^2] + \left[ \frac{5 \times 4}{2} \times 64 e^{4x} \times 6x \right] + \left[ \frac{5 \times 4 \times 3}{3 \times 2} \times 16 e^{4x} \times 6 \right] + [0][0]$$

$$y^5 = 1024 e^{4x} x^3 + 1280 e^{4x} (6x^2) + 640 e^{4x} (6x) + 1600 e^{4x} (6)$$

$$y^5 = 1024 e^{4x} x^3 + 3840 e^{4x} x^2 + 3840 e^{4x} x + 960 e^{4x}$$

$$y^5 = e^{4x} (1024 x^3 + 3840 x^2 + 3840 x + 960)$$

$$1) -x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y =$$

$$+ x^2 y'$$

$$y'' = u$$

$$u^n = y^{n+2}$$

$$v = x^2$$

$$v' = 2x$$

$$v'' = 2$$

$$v''' = 0$$

$$y^n = u^n v + n u^{n-1} v' + n(n-1) u^{n-2} v'' + n(n-1)(n-2) u^{n-3} v'''$$

$$y^2 = y^{n+2} x^2 + 2x n y^{n+1}$$

$$y^n = y^{n+2} x^2 + n y^{n+1} (2x) + n(n-1) y^n (2) + 0$$

$$y^n = y^{n+2} x^2 + 2x n y^{n+1} + 0$$

$$11) \quad xy''$$

$$u = y'$$

$$u^n = y^{n+1}$$

$$v = x$$

$$v' = 1$$

$$v'' = 0$$

$$y^n = u^2 v + n v^n - u' + \frac{n(n-1)}{2!} u^{n-2} v^2$$

$$y^n = y^{n+1} x + n x^n + 0$$

$$y^n = x y^{n+1} + n x^n$$

2)  $y$

$$u = y$$

$$u^n = y^n$$

$$v = 1$$

$$v' = 0$$

$$y^n = u^n u + n u^{n-1} v'$$
$$= y^n (u) + n (y^{n-1}) (0)$$

$$y^n = y^{n+2} x^2 + n y^{n+1} 2x + (n(n-1)) y^n + 2x^{n+1} + n y^2 + y^n$$

$$y^n = x^2 (y^{n+2}) + 2x n (y^{n+1}) + (n^2 - n) y^n + 2x (y^{n+1}) + n y^2$$

$$y^n = (n - n + n + 1) y^n + (2x n + 2x) y^{n+1} + 2x^2 (y^{n+2})$$

$$0 = x^2 (y^{n+2}) + 2x n (2x) y^{n+1} + (n^2 + 1) y^n$$

$$0 = x^2 y^n (n+2) + (2n+1) 2x (n+1) + (n^2 + 1) y^n$$

$$x^2 y (n+2) + (2n+1) 2x (n+1) + (n^2 - 1) u^n = 0$$