

D $\frac{dy}{dt} + 3y = e^{-2t}$

$t=0, y=2$

$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$Y(s)(s+3) - 2 = \frac{1}{s+2}$

$Y(s)(s+3) = \frac{1}{s+2} + 2$

$Y(s)(s+3) = \frac{1+2s+4}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s+3)}$

$L^{-1} \left[\frac{2s+5}{(s+2)(s+3)} \right] = y(s)$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

when $s = -2$

$-4+5 = A(1) + 0$

$A = 1$

when $s = -3$

$-6+5 = A(0) + B(-1)$

$-1 = -B$

$B = 1$

$Y(s) = L^{-1} \left[\frac{1}{s+2} + \frac{1}{s+3} \right]$

$y(s) = e^{-2t} + e^{-3t}$

D $3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0, y=1$

$3(sY(s) - y(0)) - 6Y(s) = \frac{2}{s^2+4}$

$3sY(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$3Y(s)(s-2) - 3 = \frac{2}{s^2+4}$

$3Y(s)(s-2) = \frac{2}{s^2+4} + 3$

$\frac{2 + 3s^2 + 12}{s^2 + 4}$

$Y(s) = \frac{3s^2 + 14}{3(s-2)(s^2+4)} = \frac{3s^2 + 14}{(3s-6)(s^2+4)}$

$Y(s) = L^{-1} \left[\frac{3s^2 + 14}{(3s-6)(s^2+4)} \right]$

$= \frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$

$3s^2 + 14 = A(s^2+4) + (Bs+C)(3s-6)$

at $s = 2$

$12+14 = A(8) + (3s+C)(0)$

$26 = 8A$

$A = \frac{13}{4}$

Using the method of coefficients

$3 = A+B \dots (1)$

$3 = \frac{13}{4} + 3B$

$3B = \frac{3}{1} - \frac{13}{4}$

$\frac{12-13}{4}$

$3B = \frac{-1}{4} \quad B = \frac{-1}{12}$

$14 = 4A - 6C$

$14 = 4 \times \frac{13}{4} - 6C$

$14 = 13 - 6C$

$1 = -6C$

$C = \frac{-1}{6}$

$$y(s) = L^{-1} \left[\frac{13}{4} \times \frac{1}{s-6} + \frac{(-\frac{1}{12}s - \frac{1}{6})}{s^2+4} \right]$$

$$= L^{-1} \left[\frac{13}{12} \times \frac{1}{s-2} - \frac{1}{12} \frac{s+1}{s^2+4} \right]$$

$$\times \frac{1}{s^2+4}$$

$$= L^{-1} \left[\frac{13}{12} \times \frac{1}{s-2} \right] + L^{-1} \left[\frac{-1}{12} \frac{s}{s^2+4} \right]$$

$$+ L^{-1} \left[\frac{-1}{6 \times 2} \times \frac{2}{s^2+4} \right]$$

$$= \frac{13}{12} e^{2t} - \frac{1}{12} \cos 2t - \frac{1}{12} \sin 2t$$

$$b) \frac{dy}{dt} - 4y = 8$$

$$\text{at } t=0 \quad y=2$$

$$s y(s) - y(0) - 4y(s) = \frac{8}{s}$$

$$s y(s) - 2 - 4y(s) = \frac{8}{s}$$

$$y(s)(s-4) = \frac{8}{s} + 2$$

$$y(s)(s-4) = \frac{8+2s}{s}$$

$$y(s) = \frac{8+2s}{s(s-4)}$$

$$y(s) = L^{-1} \left[\frac{8+2s}{s(s-4)} \right] = L^{-1} \left[\frac{A}{s} + \frac{B}{s-4} \right]$$

$$\frac{8+2s}{s(s-4)} = \frac{A(s-4)}{s(s-4)} + \frac{B(s)}{s(s-4)}$$

$$8+2s = A(s-4) + B(s)$$

$$\text{When } s=4$$

$$8+8 = A(0) + 4B$$

$$16 = 4B$$

$$B = 4$$

$$\text{When } s=0$$

$$8 = -4A + 0$$

$$-4A = 8$$

$$A = -2$$

$$y(s) = L^{-1} \left[\frac{-2}{s} + \frac{4}{s-4} \right]$$

$$y(s) = -2 + 4e^{4t}$$

$$4) \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$s^2 y(s) - s y(0) - y'(0) - 2(s y(s) - y(0)) + 5y(s) = \frac{1}{s-2}$$

$$\text{at } t=0, \quad y=2, \quad y'=1$$

$$s^2 y(s) - s(2) - 1 - 2(s y(s) - 2) + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s - 1 - 2s y(s) + 4 + 5y(s) = \frac{1}{s-2}$$

$$s^2 y(s) - 2s y(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + \frac{2s}{1} + \frac{3}{1}$$

$$\frac{1 + 2s^2 - 4s + 3s - 6}{s-2}$$

$$y(s)(s^2 - 2s + 5) = \frac{2s^2 + 3s - 9}{s-2}$$

$$y(s) = \frac{2s^2 + 3s - 9}{(s-2)(s^2 - 2s + 5)}$$

$$L^{-1} \left[\frac{2s^2 + 3s - 9}{(s-2)(s^2 - 2s + 5)} \right] = \left[\frac{A}{s-2} + \frac{B+C}{s^2 - 2s + 5} \right]$$

$$2s^2 + 3s - 9 = A(s^2 - 2s + 5) + (Bs + C)$$

$$\text{At } s=2$$

$$8 + 6 - 9 = A(4 - 8 + 5) + 0$$

$$5 = A$$

Using the method of coefficient

$$2 = A + B$$

$$2 = 5 + B$$

$$B = -3$$

$$3 = -2A - 2B + C$$

$$3 = -2(5) - 2(-3) + C$$

$$3 = -10 + 6 + C$$

$$3 = -4 + C$$

$$C = -7$$

$$y(s) = \left[\frac{5}{s-2} + \frac{(-3s-7)}{s^2-2s+5} \right]$$

$$= L^{-1} \left[\frac{5}{s-2} \right] - L^{-1} \left[\frac{3s}{(s-1)^2+4} \right] -$$

$$L^{-1} \left[\frac{7}{(s-1)^2+4} \right]$$

$$= L^{-1} \left[\frac{5}{s-2} \right] - L^{-1} \left[3 \times \frac{s}{(s-1)^2+4} \right] -$$

$$L^{-1} \left[\frac{7 \times 2}{2(s-1)^2+4} \right]$$

$$= 5e^{2t} - 3e^t \cos 2t - \frac{7}{2} e^t \sin 2t$$

$$5) \frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$$

$$\text{at } t=0, y=0, y'=2$$

$$s^2 y(s) - s y(s) - y'(s) - 6(s y(s) - y(s)) + 8y(s)$$

$$s^2 y(s) = 0 - 2 - 6s y(s) + 6y(s) + 8y(s)$$

$$= \frac{1}{s-3}$$

$$s^2 y(s) - 2 - 6s y(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s)(s^2 - 6s + 8) = \frac{1}{s-3} + \frac{2}{1}$$

$$y(s)(s^2 - 6s + 8) = \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = L^{-1} \left[\frac{2s-5}{(s-3)(s-4)(s-2)} \right]$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)}{(s-3)(s-4)(s-2)}$$

$$+ \frac{C(s-3)(s-4)}{(s-3)(s-4)(s-2)}$$

$$2s-5 = A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)$$

$$\text{at } s=2$$

$$A-5 = A(0) + B(0) + C(-1)(-2)$$

$$-1 = 2C$$

$$C = -\frac{1}{2}$$

$$\text{at } s=3$$

$$6-5 = A(-1)(1) + B(0) + C(0)$$

$$1 = -A$$

$$A = -1$$

$$\text{at } s=4$$

$$8-5 = A(0) + B(0) + C(1) + C(0)$$

$$3 = 2B$$

$$B = \frac{3}{2}$$

$$y(s) = L^{-1} \left[\frac{-1}{s-3} + \frac{3}{2} \times \frac{1}{s-4} - \frac{1}{2} \times \frac{1}{s-2} \right]$$

$$y(s) = -e^{3t} + \frac{3}{2} e^{4t} - \frac{1}{2} e^{2t}$$