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mechanical Engineering

Course : ENG 381

Assignment 4

i) $(1-x^2)\,\partial^2 y/\partial x^2 - 2x\,\partial y/\partial x + 2y = 0$

$(1+x^2)y^2 - 2xy' + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \cdots$

$[y^{(2+n)} - (1-x^2) + n y^{(1+n)} \cdot (-2x) + \frac{n(n-1)}{2!} y^n \cdot (-2)]$

$+ [y^{(1+n)} \cdot -2x + n y^n - 2 + \{2y n\}] = 0$

$(1-x^2)y^{n+2} - 2xny^{n+1} - n(n-1)y^n - 2xy^{n+1} - 2ny^n + 2y^n$

$= 0$

let $x = 0$

$y^{n+2} - n(n-1)y^n - 2ny^n + 2y^n = 0$

$y^{n+2} + y^n\,[-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n\,[-n^2 + n - 2n + 2] = 0$

$y^{n+2} + y^n\,[-n^2 - n + 2] = 0$

$y^{n+2} = [-y^n] \cdot [-n^2 - n + 2]$

$n = 0\ ;\ y^2 = -y^0 \cdot 2 = -2y^0$

$n = 1\ ;\ y^3 = -y' \cdot [0] = 0$

$n = 2\ ;\ y^4 = -y^2[-4] = 4y^2 = 4[-2y^0] = -8y^0$

$n = 3\ ;\ y^5 = -y^3 \cdot [-10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4\ ;\ y^6 = -y^4 \cdot [-18] = 18y^4 = 18 \cdot 4 \cdot -2 - y^0$

$n = 5\ ;\ y^7 = -y^5[-28] = 28(y^3) = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2 y^3}{2!} + \frac{x^3 y^3}{3!} + \cdots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2)y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2)y^0$

$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!}(18)^4(-2)y^0 + \frac{x^7}{7!}(0)$

$$y = y_0' + xy_0' - x^2 y^0 - \frac{x^4}{3\times1} y^0 - \frac{x^6}{5} y^0$$

$$y = y^0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^6}{5}\right] + y_0'(x)$$

2i) $3e^{-4t} - 5e^{4t}$

$$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$$

$$= 3\left[\frac{1}{s-a}\right] - 5\left[\frac{1}{s-a}\right]$$

$$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$$

$$= \frac{3}{s+4} - \frac{5}{s-4}$$

(ii) $\sin 4t + \cos 4t$

$$L(\sin 4t + \cos 4t] = L(\sin 4t] + L(\cos 4t)$$

$$= \frac{a}{\cancel{\cos}\, s^2+a^2} + \frac{s}{s^2+a^2}$$

$$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$$

$$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16)}$$

iii) $t^2 + 2t^2 - t + 4$

$$t^n = \frac{n!}{s^{n+1}}$$

$$= \frac{3!}{s^3+1} + 2\left[\frac{2!}{s^2+1}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$$

$$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$$

(iv) $e^{-st}\cos s^t$

$$L[\cos st] = \frac{s}{s^2+a^2}$$

$$\frac{s}{s^2+s^2} = \frac{s}{s+2s}$$

$$L[e^{-2t}\cos 5t] = \frac{s+2}{[s+2]^2 + 25}$$

v) $t\sin 3t$

$$L(\sin 3t) = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$L[t\sin 3t] = -f'(s)$$

$u = 3$ $\quad \partial u/\partial s = 0$ $\quad \dfrac{v\,\partial u/\partial s - u\,\partial v/\partial s}{v^2}$

$v = s^2+9$ $\quad \partial v/\partial s = 2s$

$$= \frac{[s^2+9]0 - 3(2s)}{[s^2+9]^2} = \frac{-6s}{[s^2+4]^2}$$

$$-F'(s) = -1\left[\frac{-6s}{(s^2+4)^2}\right]$$

$$= \frac{6s}{(s^2+9)^2}$$

vi) $\dfrac{e^{-t} - e^{-2t}}{t}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L(f(t)) = e^{-t} - e^{-2t} = 1/s+a - 1/s+9$$

$$F(\sigma) = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$$\int ft \cdot L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} 1/\sigma+1 - 1/\sigma+2 \; \partial\sigma$$

$$= \int_{\sigma=s}^{\infty} 1/1+\sigma \; \partial\sigma - \int_{\sigma=s}^{\infty} 1/\sigma+2 \; \partial\sigma$$

$$\ln[\sigma+1] - \ln[\sigma+2)]_s^{\infty}$$

$$= [\ln(\sigma+1) - \ln(\sigma+2)]_s^{\infty}$$

$$= \left[\ln \frac{\sigma+1}{(\sigma+2)}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right]$$

$$= -\ln\left[\frac{s+1}{(s+2)}\right] = \ln\left[\frac{(s+2)}{s+1}\right]$$

vii) $e^{4t}\cos 2t$

$$L[\cos 2t] = \frac{s}{s^2+2^2} = \frac{s}{s^2+4}$$

$$L[e^{4t}\cos 2t] = \frac{s-4}{[s-4]^2+4}$$

viii) $t\sin 2t$

$$L[t\sin 2t] = \frac{-\partial}{\partial s}[f(s)]$$

$$F(s) = L[\sin 2t] = \frac{2}{s^2+2^2}$$

$$f(s) = \frac{2}{s^2+4}$$

$u = 2$ $\quad \partial u/\partial s = 0$

$v = s^2+4$ $\quad \partial v/\partial s = 2s$

$$\frac{v\,\partial u/\partial s - u\,\partial v/\partial s}{v^2}$$

$$= \frac{(s^2+4)\cdot 0 - 2\cdot(2s)}{(s^2+4)^2}$$

$$= \frac{-4s}{(s^2+4)^2}$$

$$L[t\sin 2t] = F'(s)$$

$$= -1\cdot\left[{}^{-4s}/(s^2+4)^2\right]$$

$$= \frac{4s}{[s^2+4]^2}$$

ix) t^3+4t^2+5

$$L[t^3+4t^2+5]$$

$$= \frac{3!}{s^{3+1}} + 4\left[\frac{2!}{s^{2+1}}\right] + \frac{5}{s}$$

$$= \frac{6}{s^4} + \frac{8}{s^3} + \frac{5}{s}$$

x) $e^{3t}(t^2+4)$

$10 + x = t^2+4$

$L(e^{3t}x]$

$L(x) = L(t^2+4)$

$= L(t^2] + L(4]$

$= \frac{2!}{s^{2+1}} + \frac{4}{s}$

$= \frac{2}{s^3} + \frac{4}{s}$

$= \frac{2!}{s^3} + \frac{4}{s}$

$L(e^{3t}] = \frac{2}{(s-3)^3} + \frac{4}{(s-3)}$

xi) $t^2 \cos t$

$L(t^2 \cos t] = t^2 L(\cos t)$

$F(s) = L(\cos t) = \frac{s}{s^2+1^2}$

$f(s) = \frac{s}{s^2+1^2}$

$F'(s) =$ $\qquad \partial u/\partial s = 1$

$u = s$ $\qquad \partial v/\partial s = 2s$

$v = s^2+1^2$

$= \frac{[s^2+1^2) - 1 - 2s(s]}{[s^2+1^2]^2}$

$= \frac{s^2+1^2-2s^2}{[s^2+1^2]^2} = \frac{-s^2+1}{[s^2+1^2]^2}$

$-F''(s) = \frac{-\partial}{ds}\left[\frac{s^2-1}{(s^2+1^2)^2}\right]$

$u = s^2-1$ $\qquad \partial u/\partial s = 2s$

$v = (s^2+1)^2$ $\qquad \partial v/\partial s = vs(s^2+1)$

$\frac{[s^2+1)^2 \cdot 2s - (s^2-1)4s^2+4s]}{[s^2+1]^2}$

$$-F''(s) = \frac{-d}{ds}\left[\frac{s^2-1}{[s^2+1^2]^2}\right]$$

$u = s^2 - 1 \qquad du/ds = 2s$

$v = [s^2+1]^2 \qquad dv/ds = 4s[s^2+1]$

$$\frac{[s^2+1]^2 \cdot 2s - [s^2-1][4s^2+4s]}{[s^2+1]^2}$$

$$= \frac{[2s^3 - 4s^3 + 2] - [4s^5 - 4s]}{[(s^2+1)^2]}$$

$$= \frac{2s^5 - 4s^3 + 2s - 4s^5 + 4s}{[(s^2+1)^2]}$$

$$= \frac{-2s^5 - 4s^3 + 6s}{s^4 + 2s^2 + 1}$$

$$-F''(s) = \frac{-d}{ds}\left[\frac{-2s^5 - 4s^8 + 6s}{s^4 + 2s^2 + 1}\right]$$

$$F''(s) = \frac{2s^5 + 4s^3 - 6s}{s^4 + 2s^2 + 1}$$

3.)

$$\frac{s-6}{(s-3)(s-4)}$$

$$L^{-1}\left[\frac{s-5}{(s-3)(s-4)}\right] = \frac{A}{(s-3)} + \frac{B}{(s-4)}$$

$$\frac{s-5}{(s-3)(s-4)} = \frac{A(s-4) + B(s-3)}{(s-3)(s-4)}$$

Assuming $s = 4$

$4 - 5 = A(4-4) + B(4-3)$

$-1 = B(1)$

$B = -1$

Assuming $s = 3$

$3 - 5 = A(3-4) + B(3-3)$

$-2 = A(-1)$

$A = 2$

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$$\frac{s-5}{(s-3)(s-4)} = \frac{2}{s-3} + \frac{-1}{s-4}$$

$$= \frac{2}{s-3} - \frac{1}{s-4}$$

$$= 2\left[\frac{1}{s-3}\right] - \left[\frac{1}{s-4}\right]$$

$$= 2e^{3t} - e^{4t}$$

ii) $\frac{2s-6}{(s-2)(s-4)}$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right]$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$$

$$2s-6 = A(s-4) + B(s-2)$$

Assuming $s=4$

$$2(4)-6 = A(4-4) + B(4-2)$$

$$8-6 = A(0) + B(2)$$

$$2 = 2B$$

$$B = 1$$

$s = 2$

$$2(2)-6 = A(2-4) + B(2-2)$$

$$4-6 = -2A + 0$$

$$-2 = -2A$$

$$A = 1$$

$$\frac{2s-6}{(s-2)(s-4)} = \frac{1}{s-2} + \frac{1}{s-4}$$

$$L^{-1}\left[\frac{2s-6}{(s-2)(s-4)}\right] = e^{2t} - e^{4t}$$

iii) $\frac{5s-8}{s(s-4)}$

$$\mathcal{L}^{-1}\left[\frac{\cancel{5s}\ 5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$$

$$5B - 8 = A(s-4) + B(s)$$

Assuming $s=4$

$$5(4) - 8 = A(4-4) + B(4)$$

$$20 - 8 = 4B$$

$$\cancel{12 = 4B}\ 12 = 4B$$

$$B = 3$$

Assuming $s = 0$

$$5(0) - 8 = A(0-4) + B(0)$$

$$-8 = -4A$$

$$A = 2$$

$$\mathcal{L}^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$$

$$= \frac{2}{s} + 3\left[\frac{1}{s-4}\right]$$

$$= 2 + 3e^{4t}$$

iv) $\dfrac{s^2 - 3s - 4}{(s-3)(s-1)^2}$

$$= F(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$$

$$A: \left.\frac{s^2 - 2s - 4}{(s-1)^2}\right|_{s=0} = \frac{3^2 - 3(3) - 4}{(3-1)^2} = -1$$

$$B = \left.\frac{s^2 - 3s - 4}{s-3}\right|_{s=1} = \frac{1^2 - 3(1) - 4}{1-3} = 3$$

$$C: \frac{d}{dc}\left[\frac{s^2 - 2s - 4}{s-3}\right]_{s=1} = \frac{(s-3)(2s-3) - (s^2 - 3s - 4)}{(s-3)^2}$$

at $s=1$

$$\frac{(1-3)(2(1)-3) - [1^2 - 3(1) - 4]}{} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$F(t) = \cancel{m} - e^{-3t} + 3t\,e^{t} + \cancel{2}e^{t}$$

$$= e^{t}[3t+2] - e^{3t}$$

v) $\dfrac{s-5}{s^2+4s+20}$

$$L^{-1}\left[\frac{s-5}{s^2+4s+20}\right] =$$

$$f(s) = \frac{s-5}{s^2+4s+4+16} = \frac{s}{(s+2)^2+16} - \frac{5}{(s+2)^2+16}$$

$$= \frac{s+2-2}{(s+2)^2+16} - \frac{5}{(s+2)^2+16} = \frac{s+2}{(s+2)^2+6} - \frac{2}{(s+2)^2+6}$$

$$= \frac{s+2}{(s+2)^2+4^2} - \frac{7}{4} \cdot \frac{4}{(s+2)^2+4^2}$$

$$F(t) = e^{-2t}\cos 4t - 7/4\, e^{-2t} \sin 4t$$

$$= e^{-2t}[\cos 4t - 7/4 \sin 4t]$$