Faluba Opeyemi Victor
15/ENG06/030
Mechanical Engr
Engineering Maths

2) $\frac{d^2y}{dx} - 4y = 10e^{3x}$

To get CF
$m^2 - 4 = 0$
$(m-2)(m+2) = 0$
$m = 2$ or $m = -2$
$\therefore y = Ae^{m_1x} + Be^{+m_2x}$
$y = Ae^{2x} + Be^{-2x}$
Assume PI
$y = Ce^{kx}$
$y = Ce^{3x}$ — (i)
$\frac{dy}{dx} = 3Ce^{3x}$ — (ii)

$\frac{d^2y}{dx^2} = 9Ce^{3x}$ — (iii)

Sub eqn (i), (ii) and (iii) into the main equation

$\frac{d^2y}{dx} - 4y = 10e^{3x}$
$9Ce^{3x} - 4(Ce^{3x}) = 10e^{3x}$
$9Ce^{3x} - 4Ce^{kx} = 10e^{3x}$
$C\cancel{e^{3x}}(9-4) = 10\cancel{e^{3x}}$
$C(5) = 10$
$C = \frac{10}{5} = 2$

$\therefore$ Sub C into eqn (i)
$y = 2e^{3x}$
$\therefore$ The general equation becomes $= CF + PF$
$\therefore y = Ae^{-2x} + Be^{2x} + 2e^{3x}$

5) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 4\sin x$

$m^2 - 2m + 1 = 0$

$m - 1m - 1m + 1 = 0$

$m(m-1) - 1(m-1) = 0$

$(m-1)(m-1) = 0$

$m = +1$ (twice)

$y = e$

$\cdot\ e^x(A + Bx)$

$\cdot\ 4\sin x = C\cos x + D\sin x$

$\frac{dy}{dx} = -(C\sin x + D\cos x)$

$\frac{d^2y}{dx^2} = -C\cos x - D\sin x$

$-C\cos x - D\sin x - 2(-(C\sin x + D\cos x)) + C\cos x + D = 4\sin x$

$\cos x(-C - 2D + C) + \sin x(-D + 2C + D) = 4\sin x$

Comparing

$2C = 4$

$\therefore C = \frac{4}{2} = \underline{\underline{2}}$

$-2D = 0$

$\therefore D = \frac{0}{-2} = 0$

$y = e^x(A + Bx) + 2\cos x$

4) $\frac{d^2y}{dx^2} + 25y = 5x^2 + 2$

$\sqrt{m^2} = \sqrt{-25}$

$m = \pm 5j$

$y = e^{0x}(A\sin 5x + B\cos 5x)$

$y = A\sin 5x + B\cos 5x$

Assumed

$PI = 5x^2 + x$

$y = Cx^2 + Dx + E$ — (i)

$\frac{dy}{dx} = 2Cx + D + 0$ — (ii)

$\frac{d^2y}{dx^2} = 2C + 0$ — (iii)

Sub eq^n ~~into~~ (i) (ii) & (iii) into the main eq^n.

$\frac{d^2y}{dx^2} + 25y = 5x^2 + x$

$2C + 25(Cx^2 + Dx + E) = 5x^2 + x$

$25Cx^2 + 25Dx + 25E + 2C = 5x^2 + x$

Comparing variables

$25C = 5$

$C = \frac{5}{25} = \frac{1}{5}$

$25D = 1$

$D = \frac{1}{25}$

$25E + 2C = 0$

$25E = 2C$

$25E = 2\left(\frac{1}{5}\right)$

$E = \frac{2}{5} \times \frac{1}{25} = \frac{2}{125}$

$\frac{1}{5}x^2 + \frac{1}{25}x + \frac{2}{125}$

$y = A\sin 5x + B\cos 5x + \frac{x^2}{5} + \frac{x}{25} + \frac{2}{125}$

7) $\frac{3d^2y}{dx^2} - \frac{2dy}{dx} - y = 2x-3$

$3m^2-2m-1=0$

$3m^2-3m+m-1=0$

$\cancel{3m(m-1)+(m-1)=0}$

$(3m+3)(m-1)=0$

$3m=-\frac{3}{3}$ or $m=1$

$m=\frac{-3}{3}=-1$ $\quad m=1$

$y=Ae^x+Be^{-1/3x}$

Assumed PI $= Cx+D$ — (i)

$\frac{dy}{dx}=C+0$ — (ii)

$\frac{d^2y}{dx^2}=0$ — (iii)

Sub eqn (i) (ii) & (iii) into the main eqn

$3\frac{d^2y}{dx^2} - \frac{2dy}{dx} - y = 2x-3$

$3(0)-2(c)-(Cx+D)=2x-3$

$0-2c-Cx-D=2x-3$

$-2c-Cx-D=2x-3$

Comparing like terms

$-C=2$

$\therefore C=-2$

$-2c-D=-3$

$+(2C+D)=+3$

$2c+D=3$

$2(-2)+D=3$

$-4+D=3$

$D=3+4=7$

$y=Ae^x+Be^{[illegible]x}-2x+7$

1) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

Solution

$m^2 - m - 2 = 0$

$m^2 - 2m + m - 2 = 0$

$m(m-2) + 1(m-2) = 0$

$(m+1)(m-2) = 0$

$m = -1$ or 2

$y = Ae^{-x} + Be^{2x}$

PI

Assume PI : $y = c$ —— (i)

$\frac{dy}{dx} = 0$ —— (ii)

$\frac{d^2y}{dx^2} = 0$ —— (iii)

Sub eqn (i), (ii), (iii) into the original eqn

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 8$

$0 - 0 - 2c = 8$

$\therefore c = \frac{8}{-2}$

$c = -4$

$\therefore y = -4$

The general solution thus becomes

$y = CF + PI$

$y = Ae^{-x} + Be^{2x} - 4$

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

Solution

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$(m+1)(m+1) = 0$

$m = -1$ (twice)

$y = e^{x}(A + Bx)$

PI

$y = Ce^{-2x}$ —— (i)

$\frac{dy}{dx} = -2Ce^{-2x}$ —— (ii)

$\frac{d^2y}{dx^2} = +4Ce^{-2x}$ —— (iii)

Substituting (i), (ii), (iii) into eqn of the question

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-2x}$

$4Ce^{-2x} + 2(-2Ce^{-2x}) + Ce^{-2x} = e^{-2x}$

$\cancel{4Ce^{-2x}} - \cancel{4Ce^{-2x}} + Ce^{-2x} = e^{-2x}$

$Ce^{-2x} = e^{-2x}$

$C = 1$

$PF = y = e^{-2x}$

The general solution becomes

$y = e^{-x}(A + Bx) + e^{-2x}$

(4) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

Solution

$m^2 + 4m + 5 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2}$

$m = \frac{-4 \pm \sqrt{16 - 20}}{2}$

$m = \frac{-4 \pm 2j}{2} = -2 \pm j$

$y = e^{-2x}(A\cos x + B\sin x)$

Assume PI

$y = Ce^{-2x}$ —— (i)

$\frac{dy}{dx} = -2Ce^{-2x}$ —— (ii)

$\frac{d^2y}{dx^2} = 4Ce^{-2x}$ —— (iii)

Sub eqn (i), (ii), (iii) into the original equation

$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 2e^{-2x}$

$4Ce^{-2x} + 4(-2Ce^{-2x}) + 5(Ce^{-2x}) = 2e^{-2x}$

$4Ce^{-2x} - 8Ce^{-2x} + 5Ce^{-2x} = 2e^{-2x}$

$4c - 8c + 5c = 2$

$C = 2$

$y = 2e^{-2x}$

General solution becomes

$y = e^{-2x}(A\cos x + B\sin x) + 2e^{-2x}$

but when $x = 0$; $y = 1$

$1 = e^{-2(0)}(A\cos(0) + B\sin(0)) + 2e^{-2(0)}$

$1 = A + 2$

$A = -1$

when $x = 0$; $\frac{dy}{dx} = -2$

$\frac{dy}{dx} = e^{-2x}(-A\sin x + B\cos x) - 2e^{-2x}(A\cos x + B\sin x) - 4e^{-2x}$

$-2 = e^{0}(-A\sin 0 + B\cos 0) - 2e^{0}(A\cos 0 + B\sin 0) - 4e^{0}$

$-2 = B - 2A - 4$

$4 - 2 + 2A = B$

$2 + 2(-1) = B$

$0 = B$

$\therefore B = 0$

The equation becomes

$y = e^{-2x}(-\cos x) + 2e^{-2x}$

8) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

Sol:

$m^2 - 6m + 8 = 0$

$m^2 - 4m - 2m + 8 = 0$

$(m-4)(m-2) = 0$

$m = 4$ or 2

$y = Ae^{2x} + Be^{4x}$

Assume PI

$y = cxe^{4x}$ — (i)

$\frac{dy}{dx} = 4cxe^{4x} + ce^{4x}$ — (ii)

$\frac{d^2y}{dx^2} = 16cxe^{4x} + 4ce^{4x} + 4ce^{4x}$

$\frac{d^2y}{dx^2} = 16cxe^{4x} + 8ce^{4x}$ — (iii)

Substituting (i), (ii), (iii) into the main equation

$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 8e^{4x}$

$16cxe^{4x} + 8ce^{4x} - 6(4cxe^{4x} + ce^{4x}) + 8(cxe^{4x}) = 8e^{4x}$

$16cxe^{4x} + 8ce^{4x} - 24cxe^{4x} - 6ce^{4x} + 8cxe^{4x} = 8e^{4x}$

e^{4x}.

$24cxe^{4x} - 24cxe^{4x} + 2ce^{4x} = 8e^{4x}$

$2ce^{4x} = 8e^{4x}$

$c = \frac{8}{2} = 4$

$\therefore y = 4xe^{4x}$

The general solution becomes

$y = Ae^{2x} + Be^{4x} + 4xe^{4x}$