

OLATUNJI TEMITOPE OLATUNDE

15ENG02/043

COMP ENGR

1. $\frac{dy}{dt} + 3y = e^{-2t}$

$$y(t) + 3y(t) = e^{-2t}$$

$$\mathcal{L}[y(t)] = \mathcal{L}[y(t)] - Y(s)$$

$$\mathcal{L}[y(t)] = Y(s)$$

$$\mathcal{L}^{-1}[e^{-2t}] = \frac{1}{s+2}$$

$$3Y(s) - Y(s) - 3Y(s) = \frac{1}{s+2}$$

$$(s-3)Y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$2(-2) + 5 = A(-2-3) \Rightarrow A = 1/-5$$

$$2(3) + 5 = B(3+2) \Rightarrow B = 11/5$$

$$Y(s) = -\frac{1}{5}(s+2) + \frac{11}{5}(s-3)$$
$$= -\frac{1}{5}e^{-2t} + \frac{11}{5}e^{3t}$$

$$2. \quad 3 \frac{dy}{dt} - 6y = \sin 2t = 3y(t) - y(t)$$

$$2 \mathcal{L}[y(t)] = 5Y(s) - Y(s)$$

$$\mathcal{L}[y(t)] = Y(s)$$

$$\mathcal{L}[\sin 2t] = \frac{2}{s^2 + 2^2} = \frac{2}{s^2 + 4}$$

$$3s + Y(s) - 3Y(s) - 6Y(s) = \frac{2}{s^2 + 4}$$

$$Y(s) [3s - 6] = \frac{2}{s^2 + 4} + 3 = \frac{2 + 3s^2 + 12}{s^2 + 4}$$

$$= \frac{3s^2 + 14}{s^2 + 4}$$

$$Y(s) = \frac{3s^2 + 14}{(3s - 6)(s^2 + 4)} = \frac{A}{3s - 6} + \frac{B}{s^2 + 4}$$

$$3s^2 + 14 = A(s^2 + 4) + B(3s - 6)$$

$$4A - 6B = 14$$

$$B = -\frac{1}{3}$$

$$Y(s) = \frac{3}{3(s-2)} = \frac{1}{s-2}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

$$3. \quad \frac{dy}{dt} - 4y = 8$$

$$Y'(s) = 4Y(s) = 8$$

$$Y'(s) = 4Y(s) = 8$$

$$sY(s) - Y(s) - 4Y(s) = 8/s$$

$$(s-4)Y(s) = \frac{8}{s+2} = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4)$$

$$2(4)+8 = A(4-4) + B(4) \Rightarrow B = 0$$

$$L^{-1} \left[\frac{2}{s} + \frac{4}{s-4} \right] = -2 + 4e^{4t}$$

$$4. \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dt} + 5y = e^{2t}$$

$$Y''(s) - 2Y'(s) + 5Y(s) = e^{2t}$$

$$2[Y''(s)] = 5Y(s) - 5Y(0) - Y'(0)$$

$$L[Y'(s)] = 5Y(s) - Y(s)$$

$$\mathcal{L}[y(t)] = Y(s)$$

$$s^2 y(s) - s y(s) - y'(s) - 2s y(s) +$$

$$2y(s) + 5y(s) = \frac{1}{s-2}$$

$$(s^2 - 2s + 5) Y(s) + 2 - 5 Y(s) - Y'(s)$$

$$= \frac{1}{s-2}$$

$$[s^2 - 2s + 5] Y(s) = \frac{1}{s-2} - (2-5) Y(s)$$

$$= \frac{1}{s-2} + 2s - 4 + 1$$

$$= \frac{1}{s-2} + 2s - 3 = \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 3s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 3s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -3$$

$$Y(s) = \frac{2}{s-2} - \frac{3}{s^2-2s+5}$$
$$= 3e^{2t} - \frac{7}{3} + \sin 2t.$$