15/ENG05/006
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MECHATRONICS ENGR.

ENG 381 ASSIGNMENT

1) $\frac{dy}{dt} + 3y = e^{-2t}$ (Find the solution using Laplace Transform)

$y'(t) + 3y(t) = e^{-2t}$

$L[y'(t)] = sY(s) - y(0)$

$L^{-1}[e^{-2t}] = \frac{1}{s+2}$

$sY(s) - y(0) + 3Y(s) = \frac{1}{s+2}$

$(s+3)Y(s) - 2 = \frac{1}{s+2}$

$(s+3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$

$Y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3) + B(s+2)}{(s+2)(s+3)}$

$2s+5 = A(s+3) + B(s+2)$

when $s = -3$

$2(-3)+5 = A(0) + B(-3+2)$

$-B = -1$

$B = 1$

when $s = -2$

$2(-2)+5 = A(-2+3) + B(0)$

$1 = A$

$A = 1$

$Y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right] = e^{-2t} + e^{-3t}$

ii.) $3\frac{dy}{dt} - 6y = \sin 2t \Rightarrow 3y'(t) - 6y(t) = \sin 2t$

$L[y'(t)] = sY(s) - y(0)$

$L[y(t)] = Y(s)$

$L[\sin 2t] = \frac{2}{s^2+2^2} = \frac{2}{s+4}$

$3s + Y(s) - 3y(0) - 6Y(s) = \frac{2}{s^2+4}$

$Y(s)[3s-6] = \frac{2}{s^2+4} + 3 = \frac{2+3s^2+12}{s^2+4}$

$= \frac{3s^2+14}{s^2+4}$

$Y(s) = \frac{3s^2+14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$

$3s^2+14 = A(s^2+4) + B(3s-6)$

$4A - 6B = 14$

$6B = 14 - 12$

$B = -1/3$

$Y(s) = \frac{3}{3(s-2)} - \frac{1}{3(s^2+4)}$

$= e^{2t} - \frac{1}{6}\sin 2t$

3) $\frac{dy}{dt} - 4y = 8$

$y'(t) = 4Y(s) = 8$

$y'(t) - 4y(t) = 8$

$sy(s) - y(s) - 4y(s) = 8/s$

$(s-4)Y(s) = 8/s + 2 = \frac{8+2s}{s}$

$Y(s) = \frac{2s+8}{s(s-4)}$

$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$2s+8 = A(s-4) + B(s)$

$2(0)+8 = A(0-4)$

$2(4)+8 = A(4-4) + B(4) = B = 4$

~~L^{-1}~~

$$L^{-1}\left[\frac{2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$$

4) $\frac{d^2y}{dx^2} - \frac{2dy}{dt} + 5y = e^{2t}$

$y''(t) - 2y'(t) + 5y(t) = e^{2t}$

$L[y''(t)] = sY(s) - 5y(0) - y'(0)$

$L[y'(t)] = sy(s) - Y(s)$

$L[y(t)] = Y(s)$

$s'y(s) - sy(s) - y'(s) - 2sY(s) + 2Y(s) + 5y(s) = 1/s-2$

$(s^2 - 2s + 5)y(s) + (2-5)y_0 - y'(s) = \frac{1}{s-2}$

$[s^2 - 2s + 5]Y(s) = \frac{1}{s-2} - (2-s)2+1 = \frac{1}{s-2} + 2s-4+1$

$= \frac{1}{s-2} + 2s - 3 = \frac{1 + (2s+3)(s-2)}{(s-2)}$

$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$

$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 3s + 5)}$

$2s^2 = 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 3s + 5}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$

$A = 2$

$-2A + B = -7$

$B = -3$

$$Y(s) = \frac{2}{s-2} - \frac{3}{s^2-2s+5}$$

$$= 3e^{2t} - \frac{7}{3} + \sin 2t$$