

1. $(1-x^2) \frac{dy}{dx} - 2x \frac{dy}{dx} + 2y = 0$

$(1-x^2)y'' - 2xy' + 2y = 0$

$y^n = U^n V + nU^{n-1}V' + \frac{n(n-1)U^{n-2}V''}{2!} + \dots$

$y^{(n+1)} = (1-x^2) + n y^{(1+n)} (-2x) + \frac{n(n-1)y^{(n)} \cdot (-2)}{2!} + (y^{(n+1)} - 2x)$

$\frac{dy}{dt} + 3y = e^{-2t}$

$y(t) + 3y(t) = e^{-2t}$

$\mathcal{L}[y'(t)] = 5Y(s) - Y(0)$

$\mathcal{L}[y(t)] = Y(s)$

$\mathcal{L}^{-1}[e^{-2t}] = \frac{1}{s+2}$

$5Y(s) - Y(0) + 3Y(s) = \frac{1}{s+2}$

$(5-3)Y(s) - 2 = \frac{1}{s+2}$

$(5-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1 + 2s + 4}{s+2} = \frac{2s + 5}{s+2}$

$Y(s) = \frac{2s + 5}{s^2 + 3} = \frac{2}{s^2 + 4}$

$\mathcal{F} \quad Y(s) = \frac{2s + 5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$

$2(-2) + 5 = A(-2-3) \Rightarrow A = \frac{1}{-5}$

$2(3) + 5 = B(3+2) \Rightarrow B = \frac{11}{-5}$

$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = -\frac{1}{5}e^{-2t} + \frac{11}{5}e^{3t}$

1. $3 \frac{dy}{dt} - 6y = \sin 2t = 3y(t) - 6y(t)$

$\mathcal{L}[y'(t)] = 5Y(s) - Y(0)$

$\mathcal{L}[y(t)] = Y(s)$

$\mathcal{L}[\sin 2t] = \frac{2}{s^2 + 2^2} = \frac{2}{s^2 + 4}$

$3s + Y(s) - 3Y(0) - 6Y(s) = \frac{2}{s^2 + 4}$

$Y(s) [3s - 6] = \frac{2}{s^2 + 4} + 3 = \frac{2 + 3s^2 + 12}{s^2 + 4} = \frac{3s^2 + 14}{s^2 + 4}$

$$Y(s) = \frac{3s^2 + 14}{(3s-6)(s^2+4)} = \frac{A}{3s-6} + \frac{B}{s^2+4}$$

$$3s^2 + 14 = A(s^2 + 4) + B(3s - 6)$$

$$4A - 6B = 14$$

$$B = -1/3$$

$$Y(s) = \frac{3}{3(s-2)} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

ii) $dy/dt - 4y = 8$

$$Y'(s) - 4Y(s) = 8$$

$$Y'(t) - 4y(t) = 8$$

$$sY(s) - Y(0) - 4Y(s) = 8/s$$

$$(s-4)Y(s) = 8/s + 2 = \frac{8+2s}{s}$$

$$Y(s) = \frac{2s+8}{3(s-4)}$$

$$\frac{2s+8}{3(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(0)+8 = A(0-4) \quad (A=-2)$$

$$2(4)+8 = A(4-4) + B(4) \Rightarrow B=4$$

$$L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right] = -2 + 4e^{4t}$$

iv) $d^2y/dt^2 - 2dy/dt + 5y = e^{2t}$

$$Y''(s) - 2Y'(s) + 5Y(s) = \frac{1}{s-2}$$

$$L[Y''(t)] = s^2Y(s) - 5Y(0) - Y'(0)$$

$$L[Y'(t)] = sY(s) - Y(0)$$

$$L[Y(t)] = Y(s)$$

$$s^2Y(s) - 5Y(0) - Y'(0) - 2sY(s) + 2Y(0) + 5Y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5]Y(s) + (3-5)Y(0) - Y'(0) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5]Y(s) = \frac{1}{s-2} = (3-s) \frac{1}{s-2} = \frac{1}{s-2} + \frac{2s-4}{s-2}$$

$$= \frac{1}{s-2} + \frac{2s-3}{s-2} = \frac{1 + (2s-3)(s-2)}{(s-2)}$$

$$= \frac{1 + 2s^2 - 4s - 3s + 6}{s-2}$$

$$Y(s) = \frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2 - 2s + 5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$-2A + B = -7$$

$$B = -3$$

$$Y(s) = \frac{2}{s-2} - \frac{3}{s^2 - 2s + 5}$$

$$= 2e^{2t} - \frac{1}{3} + \sin 2t$$

$$v) \quad d^2y/dt^2 - 6dy/dt + 8y = e^{3t}$$

$$3^2 Y(s) - 5Y(s) - Y'(s) - 6sY(s) + 6Y(s) + 5Y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8]Y(s) = \frac{1}{s-3} + 2 = \frac{1 + 2s - 6}{s-3}$$

$$= \frac{2s-5}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A[(s-2)(s-4)] + 3[(s-3)(s-4)] + C[(s-3)(s-2)]$$

$$2s-5 = A(s^2 - 6s + 8) + B(s^2 - 7s + 7) + C(s^2 - 5s + 6)$$

$$2(3) - 5 = A(3-2)(3-4) \Rightarrow A = 4$$

$$8(4) - 5 = C(4-3)(4-2) \Rightarrow C = 3/2$$

$$-6A - 7B - 5C = 7$$

$$-6(4) - 7B - 5(3/2) = 7$$

$$-7B = 7 + 15/2 - 6 = \frac{4 + 15 - 12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$L^{-1} \left[-\frac{1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= -e^{3t} - \frac{1}{2}e^{2t} + \frac{3}{2}e^{4t}$$