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15/SC1091002

1) $\frac{dy}{dt} + 3y = e^{-2t}$

$t=0, \ y=2$

$Sy(s) + y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) \mp 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s)(s+3) = \frac{1+2s+4}{s+2}$

$y(s) = \frac{2s+5}{(s+2)(s+3)}$

$L^{-1}\left[\frac{2s+5}{(s+2)(s+3)}\right] = y(s)$

$y(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$\frac{2s+5}{(s+2)(s+3)} = \frac{A(s+3)+B(s+2)}{(s+2)(s+3)}$

$2s+5 = \frac{A(s+3)+B(s+2)}{(s+2)(s+3)}$

When $s=-2$

$-4+5 = A(1)+0$

$A=1$

When $s=-3$

$-6+5 = A(0)+B(-1)$

$-B=1$

$B=1$

$y(s) = L^{-1}\left[\frac{1}{s+2} - \frac{1}{s+3}\right]$

$y(s) = e^{-2t} + e^{-3t}$

ii) $3\frac{dy}{dx} - 6y = \sin 2t$

at $t=0, \ y=1$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3y(0) - 6g(s) = \frac{2}{s^2+4}$

$3y(s)(s-2) - 3 = \frac{2}{s^2+4} + 3$

$\frac{2+3s^2+12}{s^2+4}$

$y(s) = \frac{3s^2+14}{3(s-2)(s^2+4)} = \frac{3s^2+14}{(3s-6)(s^2+4)}$

$y(s) = L^{-1}\left[\frac{3s^2+14}{(3s-6)(s^2+4)}\right]$

$= \frac{A}{(3s-6)} + \frac{Bs+C}{(s^2+4)}$

$3s^2+14 = A(s^2+4) + (Bs+C)(3s-6)$

at $s=2$

$12+14 = A(8) + (Bs+C)(0)$

$26 = 8A$

$A = 13/4$

Using the method of coefficients

$3 = A+B \cdots (1)$

$3 = 13/4 + 3B$

$3B = 3 - 13/4$

$\frac{12-3}{4}$ $\quad B = -1/12$

$14 = 4A - 6C$

$14 = 4 \times \frac{13}{4} - 6C$

$14 = 13 - 6C$

$1 = -6C$

$C = -1/6$

$$y(s) = L^{-1}\left[\frac{13}{4} \times \frac{1}{3s-6} + \frac{\left(-\frac{1}{12}s - \frac{1}{6}\right)}{s^2+4}\right]$$

$$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2} - \frac{1}{12}\frac{s}{s^2+4} - \frac{1}{6} \times \frac{1}{s^2+4}\right]$$

$$= L^{-1}\left[\frac{13}{12} \times \frac{1}{s-2}\right] + L^{-1}\left[\frac{-1}{12}\frac{s}{s^2+4}\right]$$

$$+ L^{-1}\left[\frac{-1}{6 \times 2} \times \frac{2}{s^2+4}\right]$$

$$= \frac{13}{12}e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{12}\sin 2t$$

3) $\frac{dy}{dt} - 4y = 8$

at $t = 0$ $y = 2$

$sy(s) - y(0) + 4y(s) = \frac{8}{s}$

$sy(s) - 2 - 4y(s) = \frac{8}{s}$

$y s(s-4) = \frac{8}{s} + 2$

$y(s)(s-4) = \frac{8+2s}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$y(s) = L^{-1}\left[\frac{8+2s}{s(s-4)}\right] = L^{-1}\left[\frac{A}{s} + \frac{B}{s-4}\right]$

$\frac{8+2s}{s(s-4)} = \frac{A(s-4) + B(s)}{s(s-4)}$

$8+2s = A(s-4) + B(s)$

When $s = 4$

$8+8 = A(0) + 4B$

$16 = 4B$

$B = 4$

When $s = 0$

$8 = -4A + 0$

$-4A = 8$

$A = -2$

$y(s) = L^{-1}\left[\frac{2}{s} + \frac{4}{s-4}\right]$

$y(s) = -2 + 4e^{t}$

iii) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0))$
$+ 5y(s) = \frac{1}{s-2}$

at $t = 0$ $y = 2$ $y' = 1$

$s^2y(s) - s(2) - 1 - 2(sy(s) - 2)$
$+ 5y(s)$

$s^2y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s)$

$s^2y(s) - 2sy(s) + 5y(s) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s + 3 = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{1}{s-2} + \frac{2s}{1} + \frac{3}{1}$

$= \frac{1 + 2s^2 - 4 + 3s - 6}{s-2}$

$y(s)(s^2 - 2s + 5) = \frac{2s^2 + 3s - 9}{s-2}$

$$y(s) = \frac{2s^2+3s-9}{(s-2)(s^2-2s+5)}$$

$$L^{-1}\left[\frac{2s^2+3s-9}{(s-2)(s^2-2s+5)}\right] = \frac{A}{s-2} + \frac{(Bs+c)}{s^2-2s+5}$$

$$2s^2+3s-9 = A(s^2-2s+5) + (Bs+c)(s-2)$$

at $s=2$

$$8+6-9 = A(4-8+5)+0$$

$$5 = A$$

using the mothods of coefficient

$$2 = A+B$$

$$2 = 5+B$$

$$B = -3$$

$$3 = -2A-2B+C$$

$$3 = -2(5)-2(-3)+C$$

$$3 = -10+6+C$$

$$3 = -4C$$

$$C = 7$$

$$y(s)\left[\frac{5}{s-2} + \frac{(-3s-7)}{s^2-2s+5}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] + L^{-1}\left[\frac{(-3s-7)}{s^2-2s+5}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[\frac{3s}{(s-1)+4}\right] - L^{-1}\left[\frac{7}{(s-1)^2+4}\right]$$

$$= L^{-1}\left[\frac{5}{s-2}\right] - L^{-1}\left[3 \times \frac{5}{(s-1)^2+4}\right] - L^{-1}\left[\frac{7}{2} \times \frac{2}{(s-1)^2+4}\right]$$

$$= 5e^{2t} - 3e^{t}\cos 2t - \frac{7}{2}e^{t}\sin 2t$$

v) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

at $t=0$ $y=0$ $y'=2$

$$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)) + 8y(s)$$

$$s^2y(s) - 0 - 2 - 6sy(s) + 6(0) + 8y(s) = \frac{1}{s-3}$$

$$s^2y(s) - 2 - 6sy(s) + 8y(s) = \frac{1}{s-3}$$

$$y(s)(s^2-6s+8) = \frac{1}{s-3} + 2$$

$$y(s)(s^2-6s+8) = \frac{1+2s-6}{s-3}$$

$$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$$

$$y(s) = L^{-1}\left[\frac{2s-5}{(s-3)(s-4)(s-2)}\right]$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{s-3} + \frac{B}{s-4} + \frac{C}{s-2}$$

$$\frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A(s-4)(s-2) + B(s-3)(s-2) + C(s-3)(s-4)}{(s-3)(s-2)(s-4)}$$

$$2s-5 = A(s-4)(s-4) + B(s-3)(s-2) + C(s-3)(s-4).$$

at $s=2$.

$$4-5 = A(0) + B(0) + C(-1)(-2)$$

$$-1 = 2C$$

$$C = -\frac{1}{2}$$

at $s=3$.

$$6-5 = A(-1)(1) + B(0) + C(0)$$

$$1 = -A$$

$$A = -1$$

$$8-5 = A(0) + B(2)(1) + C(0)$$

$$3 = 2B$$

$$B = \frac{3}{2}$$

$$y(t) = L^{-1}\left[\frac{-1}{s-3} + \frac{3}{2} \times \frac{1}{s-4} - \frac{1}{2} \times \frac{1}{s-2}\right]$$

$$y(t) = -e^{3t} + \frac{3}{2}e^{4t} - \frac{1}{2}e^{2t}.$$