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i) $\frac{dy}{dt} + 3y = e^{-2t}$ at $t=0$, $y=2$

$y(0)=2$

$sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$y(s)(s+3) - 2 = \frac{1}{s+2}$

$y(s)(s+3) = \frac{1}{s+2} + 2$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)}$

$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{1+2s+4}{(s+2)(s+3)}\right]$

$\frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$s=-2$

$1+2(-2)+4 = A(-2+3)$

$1=A$

$s=-3$

$1+2(-3)+4 = B(-3+2)$

$-1=-B \quad B=1$

$y(t) = L^{-1}\left[\frac{1}{s+2} + \frac{1}{s+3}\right]$

$y(t) = e^{-2t} + e^{-3t}$

ii) $3\frac{dy}{dt} - 6y = \sin 2t$

$3(sy(s) - y(0)) - 6y(s) = \frac{2}{s^2+4}$

$3sy(s) - 3(1) - 6y(s) = \frac{2}{s^2+4}$

$y(s)(3s-6) = \frac{2}{s^2+4} + 3$

$y(s) = \frac{\cancel{2}\,2 + 3(s^2+4)}{(s^2+4)(3s-6)}$

$y(s) = \frac{2 + 3s^2 + 12}{(s^2+4)(3s-6)}$

$y = L^{-1}\left[\frac{3s^2+14}{(s^2+4)(3s-6)}\right]$

$\frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{As+B}{s^2+4} + \frac{C}{3s-6}$

$3s^2+14 = As + B(3s-6) + C(s^2+4)$

$3s^2+14 = 3As^2 - 6As + 3Bs - 6B + Cs^2 + 4C$

$3s^2+14 = s^2(3A+C) + s(-6A+3B) - 6B + 4C$

$3B - 6A = 3$, $C = 3 - 3A$

$3B - 6A = 0$

$4C - 6B = 14$

$A = -1/12$, $B = -1/6$, $C = 13/4$

$\frac{3s^2+14}{(s^2+4)(3s+6)} = \frac{-s}{12(s^2+4)} - \frac{1}{6(s^2+4)} + \frac{13}{4(3s-6)}$

$y(t) = \frac{-1}{6}\cos 2t - \frac{1}{6}\sin 2t + \frac{13}{4}e^{2t}$

iii) $\frac{dy}{dt} - 4y = 8$

at $t=0$, $y=2$

$y(0)=2$

$sy(s) - y(0) - 4y(s) = \frac{8}{s}$

$y(s)(s-4) - 2 = \frac{8}{s}$

$y(s) = \frac{8+2s}{s(s-4)}$

$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{8+2s}{s(s-4)}\right]$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$

$s=0$

$8+2(0) = A(0-4)$

$A=-2$

$s=4$

$8+2(4) = B(4)$

$B=4$

$y(t) = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$y(t) = -2 + 4e^{4t}$

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iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

at $t=0$, $y=2$, $y'=1$

$y(0)=2$, $y'(0)=1$.

$s^2y(s) - sy(0) - y'(0) - 2(sy(s) - y(0)) + 5y(s) = \frac{1}{s-2}$

$y(s)(s^2-2s+5) - 2s - 1 + 2 = \frac{1}{s-2}$

$y(s)(s^2-2s+5) = \frac{1}{s-2} - 1 + 2s$

$y(s) = \frac{1-(s-2)+2s(s-2)}{(s-2)(s^2-2s+5)}$

$y(s) = \frac{1-s+2+2s^2-4s}{(s-2)(s^2-2s+5)} = \frac{2s^2-5s+3}{(s-2)(s^2-2s+5)}$

$y(t) = L^{-1}(y(s)) = L^{-1}\left[\frac{2s^2-5s+3}{(s-2)(s^2-2s+5)}\right]$

$\frac{2s^2-5s+3}{(s-2)(s^2-2s+5)} = \frac{A}{s-2} + \frac{Bs+C}{s^2-2s+5}$

$2s^2-5s+3 = As^2 - 2As + 5A + Bs^2 - 2Bs + Cs - 2C$

$2s^2-5s+3 = s^2(A+B) + s(-2A-2B+C) + 5A - 2C$

$A+B$

$5A - 2C = 3$

$-2A - 2B + C = -5$

$A = 1/5$, $B = 9/5$, $C = -1$

v) $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$t=0$, $y=0$ $y'=2$

$y(0)=0$, $y'(0)=2$

$s^2y(s) - sy(0) - y'(0) - 6(sy(s) - y(0)$
$+ 8y(s)) = \frac{1}{s-3}$

$s^2y(s) - s(0) - 2 - 6sy(s) + 6(0) + 8y(s)$

$y(s)(s^2-6s+8) - 2 = \frac{1}{s-3}$

$y(t) = L^{-1}[y(s)] = L^{-1}\left[\frac{2s-5}{(s-2)(s-3)(s-4)}\right]$

$\frac{2s-5}{(s-2)(s-3)(s-4)} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s-4}$

$s=2$

$2(2)-5 = A(2-3)(2-4)$

$A = -1/2$

$s=4$

$2(4)-5 = C(4-2)(4-3)$

$C = 3/2$

$y(t) = L^{-1}\left[-\frac{1}{2}\cdot\frac{1}{s-2} - \frac{1}{(s-3)} + \frac{3}{2}\cdot\frac{1}{s-4}\right]$

$y t = \frac{1}{2}(3e^{4t} - 2e^{3t} - e^{2t})$