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1 $dy/dt + 3y = e^{-2t}$ $t = 0$ $y = 2$

$sy'(t) + 3(t) = e^{-2t}$

$sX(s) - y(0) + 3X(s) = \frac{1}{s+2}$

$(s+3)X(s) - y(0) = \frac{1}{s+2}$

$(s+3)X(s) - 2 = \frac{1}{s+2}$

$(s+3)X(s) = \frac{1}{s+2} + 2$

$= \frac{1 + 2(s+2)}{s+2}$

$X(s) = \frac{1 + 2s + 4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = s+2 \big|_{s=-2} = \frac{5 + 2s}{(s+3)} = \frac{5 + 2(-2)}{-2+3} = \frac{1}{1} = 1$

$B = s+3 \big|_{s=-3} = \frac{5 + 2(-3)}{-3+2} = \frac{-1}{-1} = 1$

$= \frac{1}{s+2} + \frac{1}{s+3}$

$= e^{-2t} + e^{-3t}$

ii $3\,dy/dt - 6y = \sin 2t$ $t = 0$, $y = 1$

$3(sX(s) - y(0)) - 6X(s) = \sin 2t$

$(3s - 6)X(s) - 3y(0) = \frac{2}{s^2 + 2^2}$

$(s-6)X(s) - 3 = \frac{2}{s^2+2^2}$

$(3s-6)X(s) = \frac{2}{s^2+4} + \frac{3}{1}$

$(3s-6)X(s) = \frac{2 + 3(s^2+4)}{s^2+4} = \frac{2 + 3s^2 + 12}{s^2+4} = \frac{3s^2+14}{s^2+4}$

$X = \frac{6 + s^2}{} \quad \frac{3s^2 + 14}{}$

$$A = 3s-6\Big|_{s=6/3=2} \quad \frac{3s^2+14}{(s^2+4)} = \frac{12+14}{4+4} = 13/4$$

$$3s^2+14 = A(s^2+4) + Bs + C(3s-6)$$

$$3s^2 = As^2 + 3Bs^2$$

$$B = 3 - A/3$$

$$B = \frac{3-13/4}{3} = -1/12$$

$$14 = A4 - 6C$$

$$6C = 13/4 \times 4 - 14$$

$$C = \frac{13-14}{6}$$

$$C = -1/6$$

$$= \frac{13/4}{3s-6} + \frac{-1/12}{s^2+2^2} \frac{-1/6}{s^2+2^2}$$

$$= 13/12\, e^{2t} - 1/12 \frac{s}{s^2+2^2} - 1/6\times2 \frac{2}{s^2+2^2}$$

$$\cancel{= \frac{13}{12} e^{2t} - \frac{1}{12}\sin 2t - 1}$$

$$= 13/12\, e^{2t} - \frac{1}{12}\cos 2t - \frac{1}{6\times2}\sin 2t$$

iii $dy/dt - 4y = 8$, given $t=0, y=2$

$$sY(s) - y(0) - 4Y(s) = \frac{8}{s}$$

$$(s-4)Y(s) - 2 = 8/s$$

$$(s-4)Y(s) = \frac{8}{s} + \frac{2}{1}$$

$$(s-4)Y(s) \quad \frac{8+2s}{s}$$

$$Y(s) = \frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$A = s\Big|_{s=0} \quad \frac{8+2(0)}{0-4} = \frac{8}{-4} = -2$$

$$B = s-4\Big|_{s=4} = \frac{8+2(4)}{} = 4$$

1 iv $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$ $\quad t=0,\ y=2,\ y'=1$

$s^2 y(s) - s y(0) - y'(0) + 2(s y(s) - y(0)) + 5y(0) = \frac{1}{s-2}$

$s^2 y(s) - 2s - 1 - 2sy(s) + 4 + 5y(s) = \frac{1}{s-2}$

$y(s)(s^2 - 2s + 5) - 2s - 3 = \frac{1}{s-2}$

$= r'$

$y(s) = \frac{1}{s-2} + \frac{2s}{1} - \frac{3}{1} \quad \frac{1}{s^2-2s+5}$

$y(s) = \frac{2s^2+7s-2}{s-2} \cdot \frac{1}{s^2-2s+5} \quad \times \frac{2s^2-7s+7}{(s-2)}$

$y(s) = \frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{A}{(s-2)} + \frac{Bs+C}{(s^2-2s+5)}$

$A = s-2 \Big|_{s=2}$

A $\quad \frac{2s^2-7s+7}{2^2-2(2)+5} = \frac{1}{5}$

B $\quad 2s^2-7s+7 = A(s^2-2s+5) + Bs(s-2) + C(s-2)$

$2s^2 = As^2 + Bs^2$

$2s^2 - As^2 = Bs^2$

$2 - 1/5 = B$

$B = 9/5$

ii $\quad 7 = 5A - 2C$

$7 - 5A = -2C$

$7 - 1 = -2C$

$C = \frac{6}{-2} = -3$

$\frac{1/5}{(s-2)} + \frac{9/5}{s^2-2s+5} - \frac{3}{s^2-2s+5}$

$= \frac{1}{5}\{e^{2t} + e^{t}(9\cos 2t - \frac{3}{2}\sin 2t)\}$

V. $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$

$t = 0,\ y = 0,\ y' = 2$

$s^2 y(s) - s y(0) - y'(0) - 6(s y(s) - y(0)) + 8 y(s)$
$= \frac{1}{s-3}$

$s^2 y(s) - 2 - 6 s y(s) + 8 y(s) = \frac{1}{s-3}$

~~V $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ $t = 0,\ y' = 2$~~

$y(s)(s^2 - 6s + 8) - 2 = \frac{1}{s-3}$

$y(s) = \frac{1}{s-3} + 2 \cdot \frac{1}{s^2 - 6s + 8}$

$y(s) = \frac{1 + 2(s-3)}{s-3} \cdot \frac{1}{(s-4)(s-2)}$

$y(s) = \frac{2s-5}{(s-3)(s-4)(s-2)} = \frac{A}{(s-3)} + \frac{B}{(s-4)} + \frac{C}{(s-2)}$

$A = s-3\Big|_{s=3} \quad \frac{6-5}{(3-4)(3-2)} = \frac{1}{-1} = -1$

$B = s-4\Big|_{s=4} \quad \frac{8-5}{(4-3)(4-2)} = \frac{3}{2}$

$C = s-2\Big|_{s=2} \quad \frac{4-5}{(2-3)(2-4)} = \frac{-1}{2}$

$= \frac{-1}{s-3} + \frac{3/2}{s-4} \frac{-1/2}{s-2}$

$= -e^{3t} + 3/2\, e^{4t} - 1/2\, e^{2t}$