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15/ENG07/002

PETROLEUM ENGINEERING

ENG 381

ASSIGNMENT 4

1) $(1-x^2) \cdot \partial^2/\partial x^2 - 2x \cdot \partial y/\partial x + 2y = 0$

$(1+x^2) y^2 - 2xy' + 2y = 0$

$y^n = u^n v + n u^{n-1} v' + \frac{n(n-1)}{2!} u^{n-2} v^2 + \dots$

$[y^{(2+n)} - (1-x^2) + n y^{(1+n)} \cdot (-2x) + \frac{n(n-1) y^n}{2!} \cdot (-2)]$

$+ [y^{(1+n)} - 2x + n y^n - 2 + [2yn] = 0$

$(1-x^2) y^{n+2} - 2x\, n y^{n+1} - n(n-1) y^n - 2x y^{n+1} - 2ny^n + 2y^n = 0$

let $x = 0$

$y^{n+2} - n(n-1) y^n - 2n y^n + 2y^n = 0$

$y^{n+2} + y^n [-n(n-1) - 2n + 2] = 0$

$y^{n+2} + y^n (-n^2 + n - 2n + 2) = 0$

$y^{n+2} + y^n [-n^2 - n + 2] = 0$

$y^{n+2} = -(y^n) [-n^2 - n + 2]$

$n = 0 \quad : \quad y^2 = -y^0 \cdot 2 \quad n \quad -2y^0$

$n = 1 \quad : \quad y^3 = -y^1 - [0] = 0$

$n = 2 \quad : \quad y^4 = -y^2 [-4] = 4y^2 = 4(-2y^0) = -8y^0$

$n = 3 \quad : \quad y^5 = -y^3 [10] = 10y^3 = 10 \cdot 0 = 0$

$n = 4 \quad : \quad y^6 = -y^4 \cdot (-18) = 18 y^4 = 18 \cdot 4 \cdot -2 \cdot y^0$

$n = 5 \quad : \quad y^7 = -y^5 \cdot [-28] = 28 (y^3) = 28 \cdot 0 = 0$

$y = y^0 + xy' + \frac{x^2 y^3}{2!} + \frac{x^3 y^3}{3!} + \dots$

$y = y^0 + xy' + \frac{x^2}{2!}(-2) y^0 + \frac{x^3}{3!}(0) + \frac{x^4}{4!}(4)(-2) y^0$

$+ \frac{x^5}{5!}(0) + \frac{x^6}{6!} \cdot (18)^4 (-2) y^0 + \frac{x^7}{7!}(0)$

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$y = y_0 + xy_0' - x^2y_0 - \frac{x^4}{2}y_0 - \frac{x^4}{3\times 1}y_0 - \frac{x^8}{4}y_0$

$y = y_0\left[1 - x^2 - \frac{x^4}{3} - \frac{x^8}{5}\right] + y_0'(x)$

2.i) $3e^{-4t} - 5e^{4t}$

$= L[3e^{-4t} - 5e^{4t}] \Rightarrow L[3e^{-4t}] - L[5e^{4t}]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= 3\left[\frac{1}{s+4}\right] - 5\left[\frac{1}{s-4}\right]$

$= \frac{3}{s+4} - \frac{5}{s-4}$

ii) $\cancel{\sin 4t + \cos 4t}$

$L[\sin 4t + \cos 4t] = L[\sin 4t] + L[\cos 4t]$

$= \frac{a}{s^2+a^2} + \frac{s}{s^2+a^2}$

$= \frac{4}{s^2+4^2} + \frac{s}{s^2+4^2}$

$= \frac{4}{s^2+16} + \frac{s}{s^2+16} = \frac{4+s}{(s^2+16}$

iii) $t^2 + 2t^2 - t + 4$

$t^n = \frac{n!}{s^{n+1}}$

$= \frac{3!}{s^{3+1}} + 2\left[\frac{2!}{s^{2+1}}\right] - \left[\frac{1!}{s^{1+1}}\right] + \frac{4}{s}$

$= \frac{6}{s^4} + \frac{4}{s^3} - \frac{1}{s^2} + \frac{4}{s}$

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iv) $e^{-2t} \cos 5t$

$$L[\cos 5t] = \frac{s}{s^2+a^2}$$

$$\frac{s}{s^2+5^2} = \frac{s}{s+25}$$

$$L[e^{-2t} \cos 5t] = \frac{s+2}{[s+2]^2+25}$$

v) $t \sin 3t$

$$L(\sin 3t] = \frac{a}{s^2+a^2}$$

$$= \frac{3}{s^2+3^2} = \frac{3}{s^2+9}$$

$$L[t \sin 3t] = -f'(s)$$

$u = 3$ $\quad du/ds = 0$ $\quad \frac{v\, du/ds - u\, dv/ds}{v^2}$

$v = s^2+9$ $\quad dv/ds = 2s$

$$= \frac{[s^2+9]0 - 3(2s)}{[s^2+9]^2} = \frac{-6s}{[s^2+9]^2}$$

$$-f'(s) = -1\left[\frac{-6s}{(s^2+9)^2}\right]$$

$$= \frac{6s}{[s^2+9]^2}$$

vi] $\frac{e^{-t} - e^{-2t}}{t}$

$$L\left[\frac{e^{-t} - e^{-2t}}{t}\right]$$

$$L[f(t)] = e^{-t} - e^{-2t} = 1/s+a - 1/s+a$$

$$f\,\sigma = \frac{1}{\sigma+1} - \frac{1}{\sigma+2}$$

$\int f\,\sigma \quad L\left[\frac{f(t)}{t}\right] = \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1} - \frac{1}{\sigma+2}\, d\sigma$

$= \int_{\sigma=s}^{\infty} \frac{1}{\sigma+1}\, d\sigma - \int_{\sigma=s}^{\infty} \frac{1}{\sigma+2}\, d\sigma$

$\ln[\sigma+1] - \ln[\sigma+2)]_s^{\infty}$

$= [\ln(\sigma+1) - \ln(\sigma+2)]_s^{\infty}$

$= \left[\ln \frac{\sigma+1}{\sigma+2}\right]_s^{\infty} = \ln\left[\frac{\infty+1}{(\infty+2)} - \frac{s+1}{(s+2)}\right)$

$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{A}{s} + \frac{B}{s-4}$

$5s-8 = A(s-4) + B(s)$

Assuming $s=4$

$5(4) - 8 = A(4-4) + B(4)$

$20-8 = 4B$

$12 = 4B$

$B = 3$

Assuming $s=0$

$5(0) - 8 = A(0-4) + B(0)$

$-8 = -4A$

$A = 2$

$L^{-1}\left[\frac{5s-8}{s(s-4)}\right] = \frac{2}{s} + \frac{3}{s-4}$

$= \frac{2}{s} + 3\left[1/s-4\right]$

$= 2 + 3e^{4t}$

(iv) $\frac{s^2-3s-4}{(s-3)(s-1)^2}$

$= f(s) = \frac{A}{s-3} + \frac{B}{(s-1)^2} + \frac{C}{s-1}$

$A: \left.\frac{s^2-2s-4}{(s-1)^2}\right|_{s=0} = \frac{3^2-3(3)-4}{(3-1)^2} = 1$

$$b = \frac{s^2 - 3s - 4}{s - 3}\Bigg|_{s=1} = \frac{1^2 - 3(1) - 4}{1 - 3} = 3$$

$$c = \frac{\partial}{\partial c}\left[\frac{s^2 - 3s - 4}{s - 3}\right]_{s=1} = \frac{(s-3)(2s-3) - (s^2 - 3s - 4)}{(s-3)^2}$$

at $s = 1$

$$\frac{(1-3)(2(1)-3) - (1^2 - 3(1) - 4]}{(1-3)^2} = 2$$

$$f(s) = \frac{-1}{s-3} + \frac{3}{(s-1)^2} + \frac{2}{s-1}$$

$$f(t) = -e^{-3t} + 3t\,e^{t} + 2e^{t}$$

$$= e^{t}\,t\,3t + 2] - e^{3t}$$