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MECHANICAL ENGINEERING
ENG 381
ASS V

$\frac{dy}{dt} + 3y = e^{-2t}$

given at $t=0$, $y=2$, $y(0)=2$

Soln

$y' + 3y = e^{-2t}$

$Sy(s) - y(0) + 3y(s) = \frac{1}{s+2}$

$Sy(s) - 2 + 3ys = \frac{1}{s+2}$

$y(s)[S+3] = \frac{1}{s+2} + 2 = \frac{1+2(s+2)}{s+2} = \frac{1+2s+4}{s+4}$

$y(s) = \frac{1+2s+4}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$

$A = \frac{1+2s+4}{s+3}\Big|_{s=-2} = \frac{1+2(-2)+4}{-2+3} = 1$

$B = \frac{1+2s+4}{s+2}\Big|_{s=-3} = \frac{1+2(-3)+4}{-3+2} = 1$

$y(s) = \frac{1}{s+2} + \frac{1}{s+3}$

$y(t) = e^{-2t} + e^{-3t}$

$3\frac{dy}{dt} - 6y = \sin 2t$

at $t=0$, $y=1$, $y(s)=1$

$3y' - 6y = \sin 2t$

$3[Sy(s) - y(0)] - 6y(s) = \frac{2}{s^2+4}$

$3Sy(s) - 3y(s) - 6y(s) = \frac{2}{s^2+4}$

$$s\,y(s)[3s-6] = \frac{2}{s^2+4} + \frac{3}{1} = \frac{2+3(s^2+4)}{s^2+4}$$

$$Y(s)[3s-6] = \frac{2+3s^2+12}{(s^2+4)} = \frac{3s^2+14}{(s^2+4)}$$

$$Y(s) = \frac{3s^2+14}{(s^2+4)(3s-6)} = \frac{A+Bs}{s^2+4} + \frac{C}{3s-6}$$

$$3s^2+14 = (A+Bs)(3s-6) + C(s^2+4)$$

$$3s^2+14 = 3As - 6A + 3Bs^2 - 6Bs + Cs^2 + 4C$$

$$3 = 3B + C$$

$$3 = 3B + \frac{13}{4} \qquad \text{where } C = 13/4$$

$$3B = -1/4$$

$$B = -1/12$$

$$3A - 6B = 0$$

$$3A = 6B$$

$$3A = 6 \times -1/12 \quad \therefore A = -1/6$$

$$y(s) = \frac{-1/6 - (-1/12)s}{s^2+4} + \frac{13/4}{3s-6}$$

$$\frac{-1/6}{s^2+4} - \frac{1/12\, s}{s^2+4} + \frac{13/4}{3s-6}$$

$$= -1/6 \cdot \frac{1}{s^2+2^2} - 1/12 \frac{s}{s^2+2^2} + 13/4 \frac{1}{3(s-2)}$$

$$= -1/6 \cdot \frac{1}{2}\left[\frac{2}{s^2+2^2}\right] - \frac{1}{12}\left[\frac{s}{s^2+2^2}\right] + 13/12\left[\frac{1}{s-2}\right]$$

$$y(t) = -1/12 \sin 2t - \frac{1}{12}\cos 2t + 13/12\, e^{2t}$$

$$y(t) = 1/12[-\sin 2t - \cos 2t + 13e^{2t}$$

(iii) $\frac{dy}{dt} - 4y = 8$, $t = 0, y = 2$

Soln

$dy/dt = y' = Sy(s) - y(0)$, $y = y_s$, $8L^{-1}[t] = 8/s$

$\therefore Sy(s) - y(0) - 4y_s = \frac{8}{s}$

$= Sy(s) - 4y_s = 8/s - 2$

$= y_s(s-4) = 8/s + \frac{2s}{s}$

$y_s = 8/s \times \frac{1}{s-4}$

$y_s = \frac{8+2s}{s(s-4)}$

$\frac{8+2s}{s(s-4)} = \frac{A}{s} + \frac{B}{(s-4)}$

$8 + 2s = A(s-4) + Bs$

$A + B = 2$

$-4A = 8$

$A = -2$

$B = 2+2$

$B = 4$

$$\frac{8+2s}{s(s-4)} = \frac{-2}{s} + \frac{4}{(s-4)}$$

$L^{-1}[y_s] = L^{-1}\left[\frac{-2}{s} + \frac{4}{s-4}\right]$

$y(t) = -2 + 4e^{4t}$

4) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$, $t = 0, y = 2, y = 1$

Soln

$L\{d^2y/dt^2\} = S^2y(s) - Sy(0) - y(0)$

$L[-2\frac{dy}{dt}] = -2Sy_s + 2y(0)$

$L[5y] = 5y(s)$

$s^2 y(s) - s y_0 - y_0' - 2s y_s + 2y_0 + 5y_s = \frac{1}{s-2}$

$s^2 y(s) - 2s y_s + 5y_s - 2s - 1 + 4 = \frac{1}{s-2}$

$y(s) \cdot (s^2 - 2s + 5) = \frac{1}{s-2} + 2s - 3$

$y(s)(s^2 - 2s + 5) = \frac{(2s-3)(s-2)}{(s-2)}$

$y(s) = \frac{1 + 2s^2 - s + 6}{(s-2)(s^2 - 2s + 5)}$

$\frac{2s^2 - 7s + 7}{(s-2)(s^2 - 2s + 5)} = \frac{A}{(s-2)} + \frac{Bs + c}{(s^2 - 2s + 5)}$

$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + (Bs + c)(s-2)$

$2s^2 - 7s + 7 = As^2 - 2As + 5A + Bs^2 - 2Bs + cs - 2c$

$A + B = 2$ — ①

$-2A - 2B + c = -7$ — ②

$5A - 2c = 7$ — ③

from ①

$B = 2 - A$

from ②

$-2A - 2(2 - A) + c = -7$

$-2A - 4 + 2A + c = -7$

$c = -3$

from ③

$5A - 2c = 7$

$5A - 2(-3) = 7$

$5A = 7 - 6$

$A = 1/5$

$A + 5 = 2$

$1/5 + B = 2$

$B = 2 - 1/5$

$B = 4/5$

$$\frac{2s^2-7s+7}{(s-2)(s^2-2s+5)} = \frac{1/5}{(s-2)} + \frac{9/5 s - 3}{(s^2-2s+5)}$$

$$= \frac{1/5}{s+2} + \frac{9/5 s}{(s^2-2s+5} - \frac{3}{s^2-2s+5}$$

$$= 1/5/s+2 + 9/5\left(\frac{s-1+1}{(s-1)^2+4}\right) - 3/2\left(\frac{2}{(s-1)^2+4}\right)$$

$$L^{-1}[y(s)] = L^{-1}\left[\frac{\frac{1}{5}}{(s+2)} + 9/5\left(\frac{s-1+1\times 2/2}{(s-1)^2 (s-1)^2+2^2}\right) - 3/2\left(\frac{2}{(s-1)^2+2^2}\right)\right]$$

$$= L^{-1}\left[\frac{\frac{1}{5}}{(s-2)} + 9/5\left\{\frac{s-1}{(s-1)^2+2^2} + \frac{1}{2}\left(\frac{2}{(s-1)^2+2^2}\right) - \frac{3}{2}\left(\frac{2}{(s-1)^2+2^2}\right)\right\}\right]$$

$$y(t) = \frac{1}{5}e^{2t} + \frac{9}{5}\left[e^t\cos 2t + \frac{1}{2}e^t\sin 2t\right] - 3/2\left(e^t\sin 2t\right)$$

5 $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 8y = e^{3t}$ at $t=0, y=0, y=2$

Soln

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2y(s) - sy(0) - y'(0)$$

$$L[-6\,dy/dt) = -6[sy(s) - y_0]$$

$$L[8y] = 8y(s)$$

$$L[e^{3t}] = \frac{1}{s-3}$$

$$s^2y_s - sy_0 - y_0 - 6sy_s - 6y_0 + 8y_s = \frac{1}{s-3}$$

$$s^2y_s - 6sy_s + 8y_s - 2 = \frac{1}{(s-3)}$$

$$y(s)(s^2-6s+8) = \frac{1}{s-3} + 2$$

$$y(s)(s^2-6s+8) = \frac{1+2(s-3)}{s-3}$$

$y(s) = \frac{2s-5}{(s-3)(s^2-6s+8)}$

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{A}{(s-3)} + \frac{Bs+C}{(s^2-6s+8)}$

$2s-5 = A(s^2-6s+8) + (Bs+C)(s-3)$

$2s-5 = As^2-6As+8A+Bs^2-3Bs+Cs-3C$

$A+B=0$ —— (1)

$-6A-3B+C=2$

$8A-3C=-3$

$B=-A$ from (1)

$-6A+3A+C=2$

$-3A+C=2$ —— (*)

$8A-3C=-5$ —— 8

$9A-3C=-6$

$8A-3C=-5$

$A=-1$

$B=1$

from (*)

$C=2-3$

$C=-1$

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s^2+6s+8)}$

$\frac{2s-5}{(s-3)(s^2-6s+8)} = \frac{-1}{(s-3)} + \frac{s-1}{(s-2)(s-4)}$

$\frac{s-1}{(s-2)(s-4)} = \frac{A}{(s-2)} + \frac{B}{(s-4)}$

$S-1 = A(S-4) + B(S-2)$

$S-1 = AS - 4A + BS - 2B$

$A + B = 1 \quad \text{---} (1) \times 4$

$-4A - 2B = 1 \quad \text{---} (2)$

$-4A - 4B = -4$

$-4A - 2B = -1$

$-2B = -3$

$B = 3/2$

$A = -1/2$

$$\frac{S-1}{(S-2)(S-4)} = \frac{-\frac{1}{2}}{(S-2)} + \frac{3/2}{(S-4)}$$

$$\frac{2s-5}{(S-3)(s^2-6s+8)} = \frac{-1}{(S-3)} + \left(\frac{-1/2}{(S-2)} + \frac{3/2}{(S-4)}\right)$$

$$L^{-}(y(s)) = L^{-}\left[\frac{-1}{S-3} - \frac{1/2}{S-2} + \frac{3/2}{S-4}\right]$$

$y = -e^{3t} - 1/2\, e^{2t} + 3/2\, e^{4t}$

$y = \frac{1}{2}\left[2e^{3t} + e^{2t} - 3e^{4t}\right]$