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16/ENG07/018.
PETROLEUM ENGINEERING.
ENG 282
ASSIGNMENT

Solution:

a) Step 1: Setting up a mathematical mode of the Physical process.

Let $y_{(0)}$ = the initial population of bacteria.

Let $y(t)$ = the population of bacteria still present at any instant time, t.

By the Physical Law, the time rate of change $y'(t) = \frac{dy}{dt}$ is proportional to $y(t)$. This gives the first order ODE (Ordinary Differential Equation).

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = dt \cdot k$$

$$\int \frac{1}{y} \cdot dy = k \int dt$$

$$\ln y = k.t + c$$

$$y = e^{kt + c}$$

$$y = e^{kt} \cdot e^{c} \qquad \because e^{c} = y_0$$

$$y = y_0 \cdot e^{kt}$$

Step 2: Mathematical Solution.

The ODE models exponential growth because k is positive and has the general solution (with arbitrary constant C substituted as y_0 but definite given k)

$$y_{(t)} = y_0 \cdot e^{kt} \text{ (General solution)}$$

From the experiment, the population doubles every 5 hours in a growth medium. To determine k,

Therefore @ time, $t = 5$ hours

and $y(t) = 2 \times y(0) = 2y(0)$

$$y_{(t)} = y_0 \cdot e^{kt}$$

$$2\cancel{y_0} = \cancel{y_0} \cdot e^{k(5)}$$

$$2 = e^{5k}$$

$$\ln 2 = 5k$$

$$k = \frac{\ln 2}{5}$$

$$k = \frac{0.6931471806}{5}$$

$$k = 0.1386294361$$

$$k \simeq 0.1386$$

$$\therefore y_{(t)} = y_0 \cdot e^{0.1386t}$$

To determine y_0, using the initial condition.

Therefore, @ time, $t = 0$ hour

and $y_{(t)} = 20$

$$y_{(t)} = y_0 \cdot e^{0.1386t}$$

$$20 = y_0 \cdot e^{0.1386(0)}$$

$$20 = y_0 \cdot 1$$

$$y_0 = 20$$

Hence, the particular solution governing the process is

$$y(t) = 20 \cdot e^{0.1386t} \text{ (Particular Solution).}$$

b) With this particular solution, the model can be done and solution determined.

Theoretically, $y(t) = 20 \cdot e^{0.1386t}$

At $1\frac{1}{2}$ days

To hours

1 day $= 24$ hrs

$\frac{1}{2}$ day $= 12$ hrs

$\therefore 1\frac{1}{2}$ days $= 36$ hours

with $t = 36$ hours

$$y(t) = 20 \cdot e^{0.1386 \times 36}$$

$$y(t) = 20 \cdot 146.8776607$$

$$y(t) = 2937.553214$$

$$y(t) \simeq \underline{\underline{2938}} \text{ bacteria}$$

Therefore, the estimated population of the bacteria in $1\frac{1}{2}$ days is 2,938

e) The graph of the results obtained in Question No. 8 shows that it is an exponential growth. The growth starts from the initial correct number of bacteria and increases with time. The initial number of bacteria increases with time in hours and changes from 10 to 30 to 50 with increasing time in hours because k is positive.

Therefore, there is a direct relationship between the number of bacteria and the time of growth.