OTUKOYA ISMAIL OLUWASEUN
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MECHATRONICS
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$$\frac{dy}{dt} = ky$$

$$\frac{1}{y}\,dy = k\,dt$$

$$\int \frac{1}{y}\,dy = \int k\,dt$$

$$\ln y = kt + c$$

$$y = e^{kt+c}$$

$$y = e^{kt} \cdot e^{c}$$

$$y = e^{kt} \cdot C$$

$$y = Ce^{kt}$$

given initial condition

at $t = 0$ $y = 20$

$$20 = Ce^{k(0)}$$

$$20 = Ce^{0}$$

$\therefore$ $C = 20$

Our model becomes

$$y = 20e^{kt}$$

also y doubles every 5hrs $\therefore$ $t = 5$ $y = 40$ (i.e $t = 0, y = 20$)

$\therefore$ $40 = 20e^{k(5)}$

$$\frac{40}{20} = e^{5k}$$

$$e^{5k} = 2$$

taking ln of both sides

$$5k = \ln 2$$

$$k = \frac{\ln 2}{5} \qquad k = 0.1386$$

Therefore the model is
$y = 20e^{0.1386t}$

b) Expressing $1\frac{1}{2}$ days in hours
$24hrs \longrightarrow 1 day$
$1\frac{1}{2} days = 36hrs$ (i.e $24+12$)

therefore inputing the value in the model
$y = 20e^{0.1386(36)}$
$y = 20e^{4.9890}$
$y = 20 \times 146.87$
$y = 2937.554$

at ~~$t =$~~ $y = 10, 30$ and 50
at $y = 10$ (since the same conditions still stand)
at $t = 0$ $y = 10$
$10 = Ce^{k(0)}$
$10 = Ce^{0}$
$10 = C$
Also since y doubles every 5 hrs
$20 = 10e^{5k}$
$\frac{20}{10} = e^{5k}$
$e^{5k} = 2$
taking ln of both sides
$5k = \ln 2$
$k = \frac{\ln 2}{5}$

$k = 0.1386$
$y = 10e^{0.1386t}$

For $y = 30$

$t = 0$ and $y = 30$

$30 = Ce^{k(0)}$

$30 = ee^{0}$

$30 = C$

also since y doubles every 8 hrs

$60 = 30e^{k \cdot 5}$

$\frac{60}{30} = e^{5k}$

$e^{5k} = 2$

taking ln b.s

$5k = \ln 2$

$k = \frac{\ln 2}{5} = 0.1386$

$y = 30 e^{0.1386 t}$

for $y = 50$

at $t = 0$

$50 = Ce^{k(0)}$

$50 = c$

$\therefore 100 = 50e^{5k}$

$\frac{100}{50} = e^{5k}$

taking ln of b.s

$5k = \ln 2$

$k = \frac{\ln 2}{5}$

$k = 0.1386$

$y = 50e^{0.1386t}$