OJO OLAOLUWA JOEL
Computer Engineering
15/ENG02/041

ENG382 Assignment

If the model of a system having thermocouples measuring temperatures, $T(^\circ c)$, at its different points is given by the set of expressions in eq ①. Estimate the values of the temperatures using:

① Gauss elimination method manually

$$\begin{bmatrix} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{bmatrix} \quad \text{---} \; ①$$

Solution

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & 1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Scanned by CamScanner

$$
\left[\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
\left(2-\left(\frac{2}{1}\right)\times 1\right) & \left(-1-\left(\frac{2}{1}\right)\times 1\right) & \left(1-\left(\frac{2}{1}\right)\times -2\right) & \left(2-\left(\frac{2}{1}\right)\times 1\right) & \left(1-\left(\frac{2}{1}\right)\times 3\right) & \left(-3-\left(\frac{2}{1}\right)\times -1\right) & 20-\left(\frac{2}{1}\right)\times 4 \\
\left(1-\left(\frac{1}{1}\right)\times 1\right) & \left(3-\left(\frac{1}{1}\right)\times 1\right) & \left(-3-\left(\frac{1}{1}\right)\times -2\right) & \left(-1-\left(\frac{1}{1}\right)\times 1\right) & \left(2-\left(\frac{1}{1}\right)\times 3\right) & \left(1-\left(\frac{1}{1}\right)\times -1\right) & -15-\left(\frac{1}{1}\right)\times 4 \\
\left(5-\left(\frac{5}{1}\right)\times 1\right) & \left(2-\left(\frac{5}{1}\right)\times 1\right) & \left(-1-\left(\frac{5}{1}\right)\times -2\right) & \left(-1-\left(\frac{5}{1}\right)\times 1\right) & \left(2-\left(\frac{5}{1}\right)\times 3\right) & \left(1-\left(\frac{5}{1}\right)\times -1\right) & -3-\left(\frac{5}{1}\right)\times 4 \\
\left(-3-\left(\frac{-3}{1}\right)\times 1\right) & \left(-1-\left(\frac{-3}{1}\right)\times 1\right) & \left(2-\left(\frac{-3}{1}\right)\times -2\right) & \left(3-\left(\frac{-3}{1}\right)\times 1\right) & \left(1-\left(\frac{-3}{1}\right)\times 3\right) & \left(3-\left(\frac{-3}{1}\right)\times -1\right) & 16-\left(\frac{-3}{1}\right)\times 4 \\
\left(4-\left(\frac{4}{1}\right)\times 1\right) & \left(3-\left(\frac{4}{1}\right)\times 1\right) & \left(1-\left(\frac{4}{1}\right)\times 2\right) & \left(-6-\left(\frac{4}{1}\right)\times 1\right) & \left(-3-\left(\frac{4}{1}\right)\times 3\right) & \left(-2-\left(\frac{4}{1}\right)\times -1\right) & 27-\left(\frac{4}{1}\right)\times 4
\end{array}\right]
$$

$$
= \left[\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 2 & -1 & -2 & -1 & 2 & -19 \\
0 & -3 & 9 & -6 & -13 & 6 & -23 \\
0 & 2 & -4 & 6 & 10 & 0 & 28 \\
0 & -1 & 9 & -10 & -15 & 2 & -43
\end{array}\right]
$$

$$
\left[\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & \left(2-\left(\frac{-2}{3}\right)\times -3\right) & \left(-1-\left(\frac{-2}{3}\right)\times 5\right) & \left(-2-\left(\frac{-2}{3}\right)\cdot 1\right) & \left(-1-\left(\frac{-2}{3}\right)\times -5\right) & \left(2-\left(\frac{-2}{3}\right)\times -1\right) & -19-\left(\frac{-2}{3}\right)\times 12 \\
0 & \left(-3-\left(\frac{-3}{-3}\right)\times -3\right) & \left(9-\left(\frac{3}{3}\right)\times 5\right) & \left(-6-\left(\frac{3}{3}\right)\cdot 1\right) & \left(-13-\left(\frac{3}{3}\right)\times -5\right) & \left(6-\left(\frac{3}{3}\right)\times -1\right) & -23-\left(\frac{3}{3}\right)\times 12 \\
0 & \left(2-\left(\frac{-2}{3}\right)\times -3\right) & \left(-4-\left(\frac{-2}{3}\right)\times 5\right) & \left(6-\left(\frac{-2}{3}\right)\times 1\right) & \left(10-\left(\frac{-2}{3}\right)\cdot 5\right) & \left(0-\left(\frac{-2}{3}\right)\cdot -1\right) & 28-\left(\frac{-2}{3}\right)\cdot 12 \\
0 & \left(-1-\left(\frac{-1}{3}\right)\times -3\right) & \left(9-\left(\frac{-1}{3}\right)\times 5\right) & \left(-10-\left(\frac{-1}{3}\right)\cdot 1\right) & \left(-15-\left(\frac{-1}{3}\right)\cdot 5\right) & \left(2-\left(\frac{-1}{3}\right)\cdot -1\right) & -43-\left(\frac{-1}{3}\right)\cdot 12
\end{array}\right]
$$

$$
= \left[\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\
0 & 0 & 4 & -6 & -8 & 7 & -35 \\
0 & 0 & -0.67 & 6 & 6.667 & -0.67 & 36 \\
0 & 0 & 7.33 & -10 & -13.33 & 2.33 & -47
\end{array}\right]
$$

Scanned by CamScanner

$$
= \left[\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\
0 & 0 & \left(4-\left(\frac{4}{2.33}\right)\times 2.33\right) & \left(-6-\left(\frac{4}{2.33}\right)\times -2\right) & \left(-4.33-\left(\frac{4}{2.33}\right)\times -4.33\right) & \left(7-\left(\frac{4}{2.33}\right)\times 1.33\right) & -35-\left(\frac{4}{2.33}\right)\cdot -11 \\
0 & 0 & \left(-0.67-\left(\frac{-0.67}{2.33}\right)\times 2.33\right) & \left(6-\left(\frac{-0.67}{2.33}\right)\times -2\right) & \left(6.67-\left(\frac{0.67}{2.33}\right)\times 4.3\right) & \left(-0.67-\left(\frac{0.67}{2.33}\right)\times 1.3\right) & 36-\left(\frac{0.67}{2.33}\right)\times -11 \\
0 & 0 & \left(7.33-\left(\frac{7.33}{2.33}\right)\times 2.33\right) & \left(-10-\left(\frac{7.33}{2.33}\right)\cdot -2\right) & \left(-13.3-\left(\frac{7.33}{2.33}\right)\cdot -2\right) & \left(2.33-\left(\frac{7.33}{2.33}\right)\times 1.3\right) & -47\left(\frac{7.33}{2.33}\right)\times 11
\end{array}\right]
$$

$$
= \left[\begin{array}{cccccc|c}
1 & 1 & -2 & 1 & 3 & -1 & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & 12 \\
0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\
0 & 0 & 0 & -2.57 & -0.57 & 4.71 & -16.14 \\
0 & 0 & 0 & 5.42 & 5.42 & -0.28 & 32.85 \\
0 & 0 & 0 & -3.71 & 0.28 & -1.85 & -12.43
\end{array}\right]
$$

$\cancel{a_{54} = 5.42 - \left(\frac{-2.57}{5.42}\right) \times 2}$

$a_{54} = 5.42 - \left(\frac{-5.42}{2.57}\right) \times -2.57 = 0$

$a_{55} = 5.42 - \left(\frac{-5.42}{2.57}\right) \times -0.57 = 4.22$

$a_{56} = -0.28 - \left(\frac{-5.42}{2.57}\right) \times 4.71 = 9.67$

$b_5 = 32.85 - \left(\frac{-5.42}{2.57}\right) \times -16.14 = -1.22$

$a_{64} = -3.71 - \left(\frac{-3.71}{-2.57}\right) \times -2.57 = 0$

$a_{65} = 0.28 - \left(\frac{3.71}{2.57}\right) \times -0.57 = +1.11$

$a_{66} = -1.85 - \left(\frac{3.71}{2.57}\right) \times 4.71 = -8.67$

$b_6 = -12.43 - \left(\frac{3.71}{2.57}\right) \times -16.14 = 10.89$

Scanned by CamScanner

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & -16.14 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 & -1.22 \\ 0 & 0 & 0 & 0 & 1.11 & -8.67 & 10.89 \end{array}\right]$$

$$a_{65} = 1.11 - \left(\frac{1.11}{4.22}\right) \cdot 4.22 = 0$$

$$b_6 = 10.89 - \left(\frac{1.11}{4.22}\right) \times -1.22 = 11.21$$

$$a_{66} = -8.67 - \left(\frac{-8.67}{4.22}\right) \times 9.67 = -11.21$$

$$= \left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & -16.14 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 & -1.22 \\ 0 & 0 & 0 & 0 & 0 & -11.21 & 11.21 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 \\ 0 & 0 & 0 & 0 & 0 & -11.21 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.14 \\ -1.22 \\ 11.21 \end{bmatrix}$$

Scanned by CamScanner

$$-11.21 \times T_6 = 11.21$$

$$T_6 = \frac{11.21}{11.21}$$

$$T_6 = -1$$

(ii) $4.22T_5 + 9.67T_6 = -1.22$

$$4.22T_5 + 9.67(-1) = -1.22$$

$$T_5 = \frac{-1.22 + 9.67}{4.22}$$

$$T_5 = 2$$

(iii) $-2.57T_4 - 0.57T_5 + 4.71T_6 = -16.14$

$$-2.57T_4 = -16.14 + 0.57(2) + 4.71(-1)$$

$$T_4 = \frac{-16.14 + 0.57(2) + 4.71}{-2.57}$$

$$T_4 = 4$$

(iv) $2.33T_3 - 2T_4 - 4.33T_5 + 1.33T_6 = -11$

$$2.33T_3 = -11 + 2T_4 + 4.33T_5 - 1.33T_6$$

$$T_3 = \frac{-11 + 2(4) + 4.33(2) - 1.33(-1)}{2.33}$$

$$T_3 = 3$$

(v) $-3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$

$$T_2 = \frac{12 - 5T_3 - 5T_5 - T_6}{-3}$$

$$T_2 = \frac{12 - 5(3) - 5(2) - (-1)}{-3} = -2$$

Scanned by CamScanner

$\therefore T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$

$T_1 = 4 - T_2 + 2T_3 - T_4 - 3T_5 + T_6$

$$T_1 = 4 - (-2) + 2(3) - 4 - 3(2) + (-1)$$
$$= 4 + 2 + 6 - 4 - 6 - 1$$
$$= 1$$

$$\therefore \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

$\therefore T_1 = 1$

$T_2 = -2$

$T_3 = 3$

$T_4 = 4$

$T_5 = 2$

$T_6 = 1$

Scanned by CamScanner