NELSON-UDU A. ELAH
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MTH 282 PETROLEUM ENGINEERING

Answers

c) Step 1 : Set up mathematical mode for the physical process

let $y_{(0)}$ = initial population of bacteria
let $y(t)$ = the population of bacteria still present at any instant time t.

By the physical law, the time rate of change $y(t)$ = dy/dt is proportional to $y(t)$ which gives first Order ordinary differential Equation.

$$\frac{dy}{dt} = ky$$

$$\frac{dy}{y} = dt \cdot k$$

$$\int \frac{1}{y} \cdot dy = k \int dt$$

$$\ln y = k \cdot t + c$$

$$y = e^{kt + c}$$

$$y = e^{kt} \cdot e^{c}$$

$$e^{c} = y_0$$

$$\therefore y = y_0 \cdot e^{kt}$$

Step 2 : Set up mathematical solution

The ODE models exponential growth because k is positive and has the general solution (with arbituary constant (subst. as y but depend given t)

$$y_{(t)} = y_0 \cdot e^{kt} \text{ (General solution)}$$

The population doubles every 5 hours in a growth medium to determine k

i. time (t) = 5 hours

$$y_{(t)} = 2 \times y_{(0)} = 2y_{(0)}$$

$$2y_0 = y_0 \cdot e^{k(5)}$$
$$2 = e^{5k}$$
$$\log 2 = 5k$$
$$\therefore k = \frac{\ln 2}{5}$$

$$k = \frac{0.6931471}{5}$$

$$k = 0.1386$$
$$\simeq 0.14$$

$$\therefore y(t) = y_0 \cdot e^{0.1386t}$$

determining y_0 using initial condition given
time $t = 0$ hour
$$y(t) = 20$$
$$y(t) = y_0 \cdot e^{0.1386t}$$
$$20 = y_0 \cdot e^{0.1386(0)}$$
$$20 = y_0 \cdot 1$$
$$y_0 = 20$$

$$\therefore y(t) = 20 \cdot e^{0.1386t} \text{ (particular solution)}$$

b) Using the particular solution above
$$y(t) = 20 \cdot e^{0.1386t}$$

a $1\frac{1}{2}$ days $= 24 + 12$ hours
$= 36$ hours

$\therefore t = 36$ hours
$$y(t) = 20 \cdot e^{0.1386 \times 36}$$
$$y(t) = 20 \times 146.878$$
$$= 2937.56$$
$$y(t) \simeq 2938 \text{ bacteria}$$

It can therefore be concluded that the population of bacteria in $1\frac{1}{2}$ days is 2,938.

C It can therefore be said from the graph & the results obtained that it is an exponential growth. This is because the growth starts from the initial and increases with time in hours. And so there is a direct relationship between the number of bacteria with time & growth.