Eng 282

An experiment carried out by a biomedical Engineer using a certain type of bacteria that doubles in population every 5 hours in a growth medium. If the experiment is commenced with 20 bacteria

Solution a

1 Develop a model for the system

$P(t)$ = Population of bacteria at time t

$P_0 = P(t=0)$ = Initial population of bacteria = 20

$$\frac{dP}{dt} = kP$$

separating the variables

$$\frac{dP}{P} = k\,dt$$ integrating both sides

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln P = kt$$ taking exponent of both sides

$$P = e^{kt+c} = e^{kt} \cdot e^{c}$$

$$e^{c} = P_0$$

$$\therefore P = P_0 e^{kt}$$

$$P = 2P_0$$ $t = 5$ hours

$$\therefore 2P_0 = P_0 e^{5k}$$ divide both sides by P_0

$$2 = e^{5k}$$ taking ln of both sides

$$\ln 2 = 5k$$

$$k = \frac{\ln 2}{5} = 0.138629 \approx 0.1386$$

$$\therefore P = 20 e^{0.1386t}$$ Particular solution