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15/ENG03/011

ENG 382

CIVIL ENGINEERING

Assignment 3

$$\begin{cases}
 T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\
 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\
 T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\
 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\
 -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\
 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27
 \end{cases}$$

Corresponding matrix

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 2 & -1 & 1 & 2 & 1 & -3 \\
 1 & 3 & -3 & -1 & 2 & 1 \\
 5 & 2 & -1 & -1 & 2 & 1 \\
 -3 & -1 & 2 & 3 & 1 & 3 \\
 4 & 3 & 1 & -1 & -3 & -2
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 20 \\
 -15 \\
 -3 \\
 16 \\
 -27
 \end{bmatrix}$$

To make $a_{21} = 0$

Row 2 - $\left(\frac{a_{21}}{a_{11}}\right)$ Row 1; But $a_{21}/a_{11} = 2$

For a_{21} : $2 - (2)1 = 0$

a_{22} : $-1 - (2)1 = -3$

a_{23} : $1 - (2)(-2) = 5$

a_{24} : $2 - (2)(1) = 0$

a_{25} : $1 - (2)3 = -5$

a_{26} : $-3 - (2)(-1) = -1$

a_{27} : $20 - (2)4 = 12$

To make $a_{31} = 0$

Row 3 - $\left(\frac{a_{31}}{a_{11}}\right)$ Row 1; But $\frac{a_{31}}{a_{11}} = 1$ (2)

For a_{31} : $1 - (1)1 = 0$

a_{32} : $3 - (1)1 = 2$

a_{33} : $-3 - (1)(-2) = -1$

a_{34} : $-1 - (1)1 = -2$

a_{35} : $2 - (1)3 = -1$

a_{36} : $1 - (1)(-1) = 2$

a_{37} : $-15 - (1)4 = -19$

To make $a_{41} = 0$

Row 4 - $\left(\frac{a_{41}}{a_{11}}\right)$ Row 1; But $\frac{a_{41}}{a_{11}} = 5$

For a_{41} : $5 - (5)1 = 0$

a_{42} : $2 - (5)1 = -3$

a_{43} : $-1 - (5)(-2) = 9$

a_{44} : $-1 - (5)(1) = -6$

a_{45} : $2 - (5)3 = -13$

a_{46} : $1 - (5)(-1) = 6$

a_{47} : $-3 - (5)4 = -23$

To make $a_{51} = 0$

Row 5 - $\left(\frac{a_{51}}{a_{11}}\right)$ Row 1; But $\frac{a_{51}}{a_{11}} = -3$

For a_{51} : $-3 - (-3)1 = 0$

a_{52} : $-1 - (-3)1 = 2$

a_{53} : $2 - (-3)(-2) = -4$

a_{54} : $3 - (-3)1 = 6$

a_{55} : $1 - (-3)3 = 10$

a_{56} : $3 - (-3)(-1)$

a_{57} : $16 - (-3)4 = 28$

To make $a_{61} = 0$

Row 6 - $\left(\frac{a_{61}}{a_{11}}\right)$ Row 1; But $\frac{a_{61}}{a_{11}} = 4$

For a_{61} : $4 - (4)1 = 0$

a_{62} : $3 - (4)1 = -1$

a_{63} : $1 - (4)(-2) = 9$

$$a_{64}: -1 - (4)1 = -5$$

$$a_{65}: -3 - (4)3 = -15$$

$$a_{66}: -2 - (4)(-1) = 2$$

$$a_{67}: -27 - (4)4 = -43$$

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Stage 1 (forward elimination)

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -5 & -15 & 2 & -43 \end{array} \right]$$

To make $a_{32} = 0$

Row 3 - (a_{32}/a_{22}) Row 2; But $a_{32}/a_{22} = -2/3$

$$\text{For } a_{32}: 2 - (-2/3)(-3) = 0$$

$$a_{33}: -1 - (-2/3)(5) = 7/3$$

$$a_{34}: -2 - (-2/3)(0) = -2$$

$$a_{35}: -1 - (-2/3)(-5) = -13/3$$

$$a_{36}: 2 - (-2/3)(-1) = 4/3$$

$$a_{37}: -19 - (-2/3)(12) = -11$$

To make $a_{42} = 0$

Row 4 - (a_{42}/a_{22}) Row 2; But $a_{42}/a_{22} = 1$

$$\text{For } a_{42}: -3 - (1)(-3) = 0$$

$$a_{43}: 9 - (1)(5) = 4$$

$$a_{44}: -6 - (1)(0) = -6$$

$$a_{45}: -13 - (1)(-3) = -8$$

$$a_{46}: 6 - (1)(-1) = 7$$

$$a_{47}: -23 - (1)(12) = -35$$

To make $a_{52} = 0$

Row 5 - (a_{52}/a_{22}) Row 2; But $a_{52}/a_{22} = -2/3$

$$\text{For } a_{52}: 2 - (-2/3)(-3) = 0$$

$$a_{53}: -4 - (-2/3)(5) = -2/3$$

$$a_{54}: 6 - (-2/3)(0) = 6$$

$$a_{55}: 10 - (-2/3)(-5) = 20/3$$

$$a_{56}: 0 - (-2/3)(-1) = -2/3$$

$$a_{57}: 28 - (-2/3)(12) = 36$$

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To make $a_{62} = 0$

Row 6 - (a_{62}/a_{22}) Row 2; Bnt $a_{62}/a_{22} = 1/3$

$$\text{For } a_{62}: -1 - (1/3)(-3) = 0$$

$$a_{63}: 9 - (1/3)(5) = 22/3$$

$$a_{64}: -5 - (1/3)(0) = -5$$

$$a_{65}: -15 - (1/3)(-5) = -40/3$$

$$a_{66}: 2 - (1/3)(-1) = 7/3$$

$$a_{67}: -43 - (1/3)(12) = -47$$

Stage 2

$$\left[\begin{array}{cccccc|c} 1 & 1 & 2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & -35 \\ 0 & 0 & -2/3 & 6 & 20/3 & -2/3 & 36 \\ 0 & 0 & 22/3 & -5 & -40/3 & 7/3 & -47 \end{array} \right]$$

To make $a_{43} = 0$

Row 4 - (a_{43}/a_{33}) Row 3; Bnt $a_{43}/a_{33} = 12/7$

$$\text{For } a_{43}: 4 - (12/7)(7/3) = 0$$

$$a_{44}: -6 - (12/7)(-2) = -18/7$$

$$a_{45}: -8 - (12/7)(-13/3) = -4/7$$

$$a_{46}: 7 - (12/7)(4/3) = 33/7$$

$$a_{47}: -35 - (12/7)(-11) = -113/7$$

To make $a_{53} = 0$

Row 5 - (a_{53}/a_{33}) Row 3; Bnt $a_{53}/a_{33} = -2/7$

$$\text{For } a_{53}: -2/3 - (-2/7)(7/3) = 0$$

$$a_{54}: 6 - (-2/7)(-2) = 38/7$$

$$a_{55}: 20/3 - (-2/7)(-13/3) = 34/7$$

$$a_{56}: -2/3 - (-2/7)(4/3) = -2/7$$

$$a_{57}: 36 - (-2/7)(-11) = 230/7$$

To make $a_{63} = 0$

Row 6 - (a_{63}/a_{33}) Row 3; But $a_{63}/a_{33} = 22/7$

For a_{63} : $22/3 - (22/7)(7/3) = 0$

a_{64} : $-5 - (22/7)(-2) = 9/7$

a_{65} : $-40/3 - (22/7)(-13/3) = 2/7$

a_{66} : $7/3 - (22/7)(4/3) = -13/7$

a_{67} : $-47 - (22/7)(-11) = -87/7$

Stage 3 (Forward Elimination)

1	1	-2	1	3	-1	1	4
0	-3	5	0	-5	-1	1	12
0	0	7/3	-2	-13/3	4/3	1	-11
0	0	0	-18/7	-4/7	33/7	1	-113/7
0	0	0	38/7	38/7	-2/7	1	230/7
0	0	0	9/7	2/7	-13/7	1	-87/7

To make $a_{54} = 0$

Row 5 - (a_{54}/a_{44}) Row 4; But $a_{54}/a_{44} = 19/9$

For a_{54} : $38/7 - (-19/9)(-18/7) = 0$

a_{55} : $38/7 - (-19/9)(-4/7) = 38/9$

a_{56} : $-2/7 - (-19/9)(33/7) = 29/3$

a_{57} : $230/7 - (-19/9)(-113/7) = -11/9$

To make $a_{64} = 0$

Row 6 - (a_{64}/a_{44}) Row 4; But $a_{64}/a_{44} = -1/2$

For a_{64} : $9/7 - (-1/2)(-18/7) = 0$

a_{65} : $2/7 - (-1/2)(-4/7) = 0$

a_{66} : $-13/7 - (-1/2)(33/7) = 1/2$

a_{67} : $-87/7 - (-1/2)(-113/7) = -41/2$

STAGE 4

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$$\left[\begin{array}{ccccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & 1 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & 1 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & 1 & -11/9 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & -41/2 \end{array} \right]$$

To make $a_{65} = 0$

Row 6 - (a_{65}/a_{55}) Row 5; But $a_{65}/a_{55} = 0$

For $a_{65} = 0 - (0) 38/9 = 0$

$$a_{66}: 1/2 - (0) (29/3) = 1/2$$

$$a_{67}: -41/2 - (0) (-11/9) = -41/2$$

Stage 5

$$\left[\begin{array}{ccccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & 5 & -1 & 1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & 1 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & 1 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & 1 & -11/9 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 1 & -41/2 \end{array} \right]$$

Back substitution

$$T_6: 0T_1 + 0T_2 + 0T_3 + 0T_4 + 0T_5 + 1/2(T_6) = -41/2$$

$$\frac{T_6}{2} = \frac{-41}{2}$$

$$T_6 = -41$$

$$T_6 = -41$$

$$T_5: 0T_1 + 0T_2 + 0T_3 + 0T_4 + 38/9 T_5 + 29/3 T_6 = -11/9$$

$$\frac{38T_5}{9} + \frac{29}{3} (-41) = \frac{-11}{9}$$

$$\frac{38T_5}{9} = \frac{3556}{9}$$

$$T_5 = \frac{3556}{38} = 93.5789$$

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$$T_4: 0T_1 + 0T_2 + 0T_3 + \left(-\frac{18}{7}\right)T_4 + \left(-\frac{4}{7}\right)T_5 + \left(\frac{33}{7}\right)T_6 = -\frac{113}{7}$$

$$\frac{-18T_4}{7} - \frac{1016}{19} - \frac{1353}{7} = -\frac{113}{7}$$

$$\frac{-18T_4}{7} = \frac{30692}{133}$$

$$T_4 = -89.6842$$

$$T_3: 0T_1 + 0T_2 + \left(\frac{7}{3}\right)T_3 + (-2)T_4 + \left(-\frac{13}{3}\right)T_5 + \left(\frac{4}{3}\right)T_6 = -11$$

$$\frac{7T_3}{3} - \left(2 \times -\frac{1704}{19}\right) - \left(\frac{13}{3} \times \frac{3556}{38}\right) + \left(\frac{4}{3} \times -41\right) = -11$$

$$\frac{7}{3}T_3 = -11 - \frac{3408}{19} + \frac{46228}{114} + \frac{164}{3}$$

$$T_3 = \frac{15879}{57} \times \frac{3}{7}$$

$$T_3 = 115.6316$$

$$T_2: 0T_1 + (-3)T_2 + (5)T_3 + (0)T_4 + (-5)T_5 + (-1)T_6 = 12$$

$$-3T_2 + 5T_3 - 5T_5 - T_6 = 12$$

$$-3T_2 + \left(5 \times \frac{46137}{399}\right) - \left(5 \times \frac{3536}{38}\right) - (-41) = 12$$

$$-3T_2 = 12 - 41 - \frac{230685}{399} + \frac{17780}{38}$$

$$T_2 = \frac{-2646}{19} \div -3$$

$$T_2 = 46.4311$$

$$T_1: T_1 + T_2 + (-2)T_3 + (1)T_4 + (3)T_5 + (-1)T_6 = 4$$

$$T_1 + \frac{882}{19} + \left(-2 \times \frac{46137}{399}\right) + \left(-\frac{1704}{19}\right) + \left(3 \times \frac{3556}{38}\right) + (-1 \times -41) = 4$$

$$T_1 = 4 - 41 - \frac{882}{19} + \frac{92274}{399} + \frac{1704}{19} - \frac{10668}{38}$$

$$T_1 = -432.105$$