

### ENG 382 Assignment III

$$\left. \begin{aligned} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 &= 4 \\ 2T_1 - T_2 + T_3 + 3T_4 + T_5 - 3T_6 &= 20 \\ T_1 + 3T_2 + 3T_3 - T_4 + 2T_5 + T_6 &= -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 &= -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 &= 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 &= -27 \end{aligned} \right\}$$

#### Forward Elimination

$$\begin{bmatrix} 1 & +1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & +3 & -3 & -1 & 2 & 1 \\ 5 & +2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & +3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Row 2

$$\begin{aligned} 2 - \left(\frac{2}{1}\right)(1) &= 0 \\ -1 - \left(\frac{2}{1}\right)(1) &= -3 \end{aligned}$$

Row 3

$$\begin{aligned} 1 - \left(\frac{1}{1}\right)(1) &= 0 \\ 3 - 1(1) &= 2 \end{aligned}$$

$$1 - (2)(-2) = 5$$

$$-3 - 1(-2) = -1$$

$$2 - (2)(1) = 0$$

$$-1 - 1(1) = -2$$

$$1 - (2)(3) = -5$$

$$2 - 1(3) = -1$$

$$-3 - (2)(-1) = -1$$

$$1 - 1(-1) = 2$$

$$20 - (2)(4) = 12$$

$$-15 - 1(4) = -19$$

Row 4

$$5 - \left(\frac{5}{7}\right)(1) = 0$$

$$2 - 5(1) = -3$$

$$-1 - 5(-2) = 9$$

$$-1 - 5(1) = -6$$

$$2 - 5(3) = -13$$

$$1 - 5(-1) = 7$$

$$-3 - 5(4) = -23$$

Row 6

$$4 - \left(\frac{4}{1}\right)(1) = 0$$

$$3 - (4)(1) = -1$$

$$1 - (4)(-2) = 9$$

$$-6 - (4)(1) = -10$$

$$-3 - (4)(3) = -15$$

$$-2 - (4)(-1) = 2$$

$$-27 - (4)(4) = -43$$

Row 5

$$-3 - (-3/1)(1) = 0$$

$$-1 + (3)(1) = 2$$

$$2 + (3)(-2) = -4$$

$$3 + (3)(1) = 6$$

$$1 + (3)(3) = 10$$

$$3 + (3)(-1) = 0$$

$$16 + (3)(4) = 28$$



1	1	-2	1	3	-1	16
0	-3	5	0	-5	-1	12
0	2	-1	-2	-1	2	-19
0	-3	9	-6	-13	6	-23
0	2	-4	6	10	0	28
0	-1	9	-10	-15	2	-43

Row 3

$$2 - \left(\frac{2}{-3}\right)(-3) = 0$$

$$-1 - \left(\frac{2}{-3}\right)(5) = 7/3$$

$$-2 - \left(\frac{2}{-3}\right)(0) = -2$$

$$-1 - \left(\frac{2}{-3}\right)(-5) = -13/3$$

$$2 - \left(\frac{2}{-3}\right)(-1) = 4/3$$

$$-19 - \left(\frac{2}{-3}\right)(12) = -11$$

Row 4

$$-3 - \left(\frac{-3}{-3}\right)(-3) = 0$$

$$4 - (1)(5) = 4$$

$$-6 - (1)(0) = -6$$

$$-13 - (1)(-5) = -8$$

$$6 - (1)(-1) = 7$$

$$-23 - (1)(12) = -35$$

Row 5

$$2 - \left(\frac{2}{-3}\right)(-3) = 0$$

$$-4 - \left(\frac{2}{-3}\right)(5) = -3/3$$

$$6 - \left(\frac{2}{-3}\right)(0) = 6$$

$$10 - \left(\frac{2}{-3}\right)(-15) = 20/3$$

$$0 - \left(\frac{2}{-3}\right)(-1) = -2/3$$

$$28 - \left(\frac{2}{-3}\right)(12) = 36$$

Row 6

$$-1 - \left(\frac{-1}{-3}\right)(-3) = 0$$

$$9 - \left(\frac{1}{3}\right)(5) = 22/3$$

$$-10 - \left(\frac{1}{3}\right)(0) = -10$$

$$-15 - \left(\frac{1}{3}\right)(-5) = -40/3$$

$$2 - \left(\frac{1}{3}\right)(-1) = 7/3$$

$$-43 - \left(\frac{1}{3}\right)(12) = -47$$



1	1	-2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	0	$\frac{7}{3}$	-2	$-\frac{13}{3}$	$\frac{4}{3}$	-11
0	0	4	-6	-8	7	-35
0	0	$-\frac{2}{3}$	6	$\frac{20}{3}$	$-\frac{2}{3}$	36
0	0	$\frac{22}{3}$	-10	$-\frac{40}{3}$	$\frac{7}{3}$	-47

Row 4

$$4 - (\frac{4}{1/3})(\frac{7}{3}) = 0$$

$$-6 - (\frac{12}{7})(-2) = -\frac{18}{7}$$

$$-8 - (\frac{12}{7})(-\frac{13}{3}) = -\frac{4}{7}$$

$$7 - (\frac{12}{7})(\frac{4}{3}) = \frac{33}{7}$$

$$-35 - (\frac{12}{7})(-11) = -\frac{3}{7}$$

Row 5

$$-\frac{2}{3} - (-\frac{2}{3/1/3})(\frac{7}{3}) = 0$$

$$6 - (-\frac{2}{7})(-2) = \frac{38}{7}$$

$$\frac{20}{3} - (-\frac{2}{7})(-\frac{13}{3}) = \frac{38}{7}$$

$$-\frac{2}{3} - (-\frac{2}{7})(\frac{4}{3}) = -\frac{2}{7}$$

$$36 - (-\frac{2}{7})(-11) = \frac{230}{7}$$

Row 6

$$\frac{22}{3} - (\frac{22}{3/7/3})(\frac{7}{3}) = 0$$

$$-10 - (\frac{22}{7})(-2) = -\frac{26}{7}$$

$$-\frac{40}{3} - (\frac{22}{7})(-\frac{13}{3}) = \frac{2}{7}$$

$$\frac{7}{3} - (\frac{22}{7})(\frac{4}{3}) = -\frac{13}{7}$$

$$-47 - (\frac{22}{7})(-11) = -\frac{87}{7}$$

1	1	-2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	0	$\frac{7}{3}$	-2	$-\frac{13}{3}$	$\frac{4}{3}$	-11
0	0	0	$-\frac{18}{7}$	$-\frac{4}{7}$	$\frac{33}{7}$	$-\frac{113}{7}$
0	0	0	$\frac{38}{7}$	$\frac{38}{7}$	$-\frac{2}{7}$	$\frac{230}{7}$
0	0	0	$-\frac{26}{7}$	$\frac{2}{7}$	$-\frac{13}{7}$	$-\frac{87}{7}$



Row 5

$$38/7 - (38/7 - 18/7)(-18/7) = 0$$

$$38/7 - (-\frac{38}{18})(-\frac{4}{7}) = \frac{38}{9}$$

$$-2/7 - (-\frac{38}{18})(33/7) = 29/3$$

$$\frac{230}{7} - (-\frac{38}{18})(-\frac{113}{7}) = -11/9$$

Row 6

$$-26/7 - (-26/7 - 18/7)(-18/7) = 0$$

$$2/7 - (13/9)(-4/7) = 10/9$$

$$-13/7 - (13/9)(33/7) =$$

$$-87/7 - (13/9)(-\frac{113}{7}) =$$

1	1	-2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	0	7/3	-2	-13/3	4/3	-11
0	0	0	-18/7	-4/7	33/7	-113/7
0	0	0	0	38/9	29/3	-11/9
0	0	0	0	10/9	-26/3	98/9

Row 6

$$80/9 - (80/9 - 38/9)(38/9) = 0$$

$$-26/3 - (\frac{10}{38})(29/3) = -213/19$$

$$98/9 - (\frac{10}{38})(-11/9) = 213/19$$

1	1	-2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	0	7/3	-2	-13/3	4/3	-11
0	0	0	-18/7	-4/7	33/7	-113/7
0	0	0	0	38/9	29/3	-11/9
0	0	0	0	0	-213/19	213/19



Back substitution.

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -213/19 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -113/7 \\ -11/9 \\ 213/19 \end{bmatrix}$$

$$T_6 \cdot -213/19 = 213/19$$

$$T_6 = -1$$

$$T_5 \cdot 38/9 + T_6 \cdot 29/3 = -11/9$$

$$T_5 = 2$$

$$T_4 \cdot -18/7 + T_5 \cdot -4/7 + T_6 \cdot 33/7 = -113/7$$

$$T_4 = 4$$

$$T_3 \cdot 7/3 + T_4 \cdot -2 + T_5 \cdot -13/3 + T_6 \cdot 4/3 = -11$$

$$T_3 = 3$$

$$T_2 \cdot -3 + T_3 \cdot 5 + T_4 \cdot 0 + T_5 \cdot -5 + T_6 \cdot -1 = 12$$

$$T_2 = -2$$

$$T_1 \cdot 1 + T_2 \cdot 1 + T_3 \cdot -2 + T_4 \cdot 1 + T_5 \cdot 3 + T_6 \cdot -1 = 4$$

$$T_1 = 1$$