$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$

$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$

$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$

$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$

$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$

$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_4 - 3T_5 - 2T_6 = -27$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

The Augmented matrix becomes

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array}\right]$$

Row 2 $- \frac{2}{1}$ Row 1 : step 1

$2 - \frac{2}{1}(1) = 0$

$-1 - \frac{2}{1}(1) = -3$

$1 - \frac{2}{1}(-2) = 5$

$2 - \frac{2}{1}(1) = 0$

$1 - \frac{2}{1}(3) = -5$

$-3 - \frac{2}{1}(-1) = -1$

$20 - \frac{2}{1}(4) = 12$

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$$\begin{array}{c|cccccc:c}
1 & 1 & -2 & 1 & 3 & -1 & & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & & 12 \\
0 & 3 & -3 & -1 & 2 & 1 & & -15 \\
5 & 2 & -1 & -1 & 2 & 1 & & -3 \\
-3 & -1 & 2 & 3 & 1 & 3 & & 16 \\
4 & 3 & 1 & -6 & -3 & -2 & & -27
\end{array}$$

Row 3 − Row 1 : step 2

$1 - 1 = 0$

$3 - 1 = 2$

$-3 + 2 = -1$

$-1 \cancel{+1} = \cancel{0}\ 2$

$2 - 3 = -1$

$1 + 1 = 2$

$-15 - 4 = -19$

$$\begin{bmatrix}
1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\
0 & 2 & -1 & -2 & -1 & 2 & : & -19 \\
5 & 2 & -1 & -1 & 2 & 1 & : & -3 \\
-3 & -1 & 2 & 3 & 1 & 3 & : & 16 \\
4 & 3 & 1 & -6 & -3 & -2 & : & -27
\end{bmatrix}$$

Row 4 − $\frac{5}{1}$ Row 1 : step 3

$5 - \frac{5}{1}(1) = 0$

$2 - \frac{5}{1}(1) = -3$

$-1 - \frac{5}{1}(-2) = 9$

$-1 - \frac{5}{1}(1) = -6$

$2 - \frac{5}{1}(3) = -13$

$1 - \frac{5}{1}(-1) = 6$

$-3 - \frac{5}{1}(4) = -23$

$$\begin{bmatrix}
1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\
0 & 2 & -1 & -2 & -1 & 2 & : & 19 \\
0 & -3 & 9 & -6 & -13 & 6 & : & -23 \\
-3 & -1 & 2 & 3 & 1 & 3 & : & 16 \\
4 & 3 & 1 & -6 & -3 & -2 & : & -27
\end{bmatrix}$$

Row 5 + $\frac{3}{1}$ Row 1 : step 4

$-3 + \frac{3}{1}(1) = 0$

$-1 + \frac{3}{1}(1) = \cancel{2}$

$2 + \frac{3}{1}(-2) = -4$

$3 + \frac{3}{1}(1) = 6$

$1 + \frac{3}{1}(3) = 10$

$3 + \frac{3}{1}(-1) = 0$

$16 + \frac{3}{1}(4) = 28$

$$\begin{bmatrix}
1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\
0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\
0 & 2 & -1 & -2 & -1 & 2 & : & 19 \\
0 & -3 & 9 & -6 & -13 & 6 & : & -23 \\
0 & 2 & -4 & 6 & 10 & 0 & : & 28 \\
4 & 3 & 1 & -6 & -3 & -2 & : & -27
\end{bmatrix}$$

Step 5 : Row 6 − $\frac{4}{1}$ Row 1

$4 - \frac{4}{1}(1) = 0$

$3 - \frac{4}{1}(1) = -1$

$1 - \frac{4}{1}(-2) = 9$

$-6 - \frac{4}{1}(1) = -10$

$-3 - \frac{4}{1}(3) = -15$

$-2 - \frac{4}{1}(-1) = \cancel{0}$

$-27 - \frac{4}{1}(4) = -43$

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$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & -43 \end{array}\right]$$

Row 3 + $\frac{2}{3}$ Row 2 : Step 6

$0 + \frac{2}{3}(0) = 0$

$2 + \frac{2}{3}(-3) = 0$

$-1 + \frac{2}{3}(5) = \frac{7}{3}$

$-2 + \frac{2}{3}(0) = -2$

$-1 + \frac{2}{3}(-5) = -\frac{13}{3}$

$2 + \frac{2}{3}(-1) = \frac{4}{3}$

$-19 + \frac{2}{3}(12) = 27$

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & -11 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & -43 \end{array}\right]$$

Row 4 + $\frac{3}{2}$ Row 3 Step 7

$-3 + \frac{3}{2}(2) = 0$

$9 + \frac{3}{2}(-1) = \frac{15}{2}$ or 7.5

$-6 + \frac{3}{2}(-2) = -9$

$-13 + \frac{3}{2}(-1) = -14.5$

$6 + \frac{3}{2}(2) = 9$

$-23 + \frac{3}{2}(-19) = -51.5$

Rows + $\frac{2}{3}$ Row 4 Step 8

$2 + \frac{2}{3}(-3) = 0$

$-4 + \frac{2}{3}(9) = 2$

$6 + \frac{2}{3}(-6) = 2$

$10 + \frac{2}{3}(-13) = \frac{4}{3}$

$0 + \frac{2}{3}(6) = 4$

$28 + \frac{2}{3}(-23) = 38/3$

Step 9

Row 6 + $\frac{1}{2}$ Row 5

$-1 + \frac{1}{2}(2) = 0$

$9 + \frac{1}{2}(-4) = 7$

$-10 + \frac{1}{2}(6) = -7$

$-15 + \frac{1}{2}(10) = -10$

$2 + \frac{1}{2}(0) = 2$

$-43 + \frac{1}{2}(28) = -29$

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & -11 \\ 0 & 0 & 7.5 & -9 & -14.5 & 9 & -51.5 \\ 0 & 0 & 2 & 2 & \frac{4}{3} & 4 & 38/3 \\ 0 & 0 & 7 & -7 & -10 & 2 & -29 \end{array}\right]$$

Row 4 − $\frac{45}{14}$ Row 3 : 10

$7.5 - \frac{45}{14}(\frac{7}{3}) = 0$

$-9 - \frac{45}{14}(-2) = -18/7$

$-14.5 - \frac{45}{14}(-\frac{13}{3}) = -4/7$

$9 - \frac{45}{14}(\frac{4}{3}) = 33/7$

$-51.5 - \frac{45}{14}(-11) = -113/7$

11 : Row 5 − $\frac{2}{7.5}$ Row 4

$2 - \frac{2}{7.5}(7.5) = 0$

$2 - \frac{2}{7.5}(-9) = 4.4$

$26/5 = \frac{4}{3} - \frac{2}{7.5}(-14.5) = 16/15$

$4 - \frac{2}{7.5}(9) = 8/5$

$38/3 - \frac{2}{7.5}(-51.5) = 26.4$

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12 Row 6 − 7/2 Row 5

$7 - \frac{7}{2}(2) = 0$

$-7 - \frac{7}{2}(2) = -14$

$-10 - \frac{7}{2}(4/3) = -44/3$

$2 - \frac{7}{2}(4) = -12$

$-29 - \frac{7}{2}(38/3) = -220/3$

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 4.4 & 26/5 & 8/5 & 26.4 \\ 0 & 0 & 0 & -14 & -44/3 & -12 & -220/3 \end{array}\right]$$

13: Row 5 + 77/45 Row 4

$4.4 + \frac{77}{45}(-18/7) = 0$

$38/9 = 26/5 + \frac{77}{45}[-4/7]$

$29/3 = 8/5 + \frac{77}{45}[33/7]$

$-11/9 = 26.4 + \frac{77}{45}[-113/7]$

14: Row 6 + 14/4.4 Row 5

$-14 + \frac{14}{4.4}[4.4] = 0$

$-44/3 + \frac{14}{4.4}[26/5] = 62/33$

$-12 + \frac{14}{4.4}[8/5] = -76/11$

$-220/3 + \frac{14}{4.4}[26.4] = 33/3$

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 62/33 & -76/11 & 33/3 \end{array}\right]$$

Row 6 − 93/209 Row 5

$\frac{62}{33} - \frac{93}{209}\left[\frac{38}{9}\right] = 0$

$\frac{-76}{11} - \frac{93}{209}\left[\frac{29}{3}\right] = -213/19$

$\frac{32}{3} - \frac{93}{209}\left[\frac{-11}{9}\right] = \frac{213}{19}$

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 38/9 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 0 & -213/19 & 213/19 \end{array}\right]$$

$-213/19\, T_6 = 213/19$

$T_6 = -1$ — ①

$38/9\, T_5 + 29/3\, T_6 = -11/9$

$38/9\, T_5 = 76/9$

$T_5 = 2 - 2$

$-18/7\, T_4 - 4/7\, T_5 + 33/7\, T_6 = -113/7$

$-18/7\, T_4 - 8/7 - 33/7 = -113/7$

$-18/7\, T_4 = -72/7$

$T_4 = 4$

$7/3\, T_3 - 2T_4 - 13/3\, T_5 + 4/3\, T_6 = -11$

$7/3\, T_3 - 8 - 26/3 + 4/3 = -11$

$7/3\, T_3 = 7$

$T_3 = 3$

$-3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$

$-3T_2 + 15 + 0 - 10 + 1 = 12$

$-3T_2 = 6$

$T_2 = -2$

$1T_1 + 1T_2 - 2T_3 + 1T_4 + 3T_5 - 1T_6 = 4$

$T_1 - 2 - 6 + 4 + 6 + 1 = 4$

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$T_1 = 1$

Therefore: $T_1 = 1$

$T_2 = -2$

$T_3 = 3$

$T_4 = 4$

$T_5 = 2$

$T_6 = -1$

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