

SALAMT ABDULLAH AEC 2.T.

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ELECT- ELECT-

Example

$$\begin{cases} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{cases}$$

the Augmented matrix

$$\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & 4 \\ (2) & -1 & 1 & 2 & 1 & -3 & T_2 & 20 \\ (1) & (3) & -3 & -1 & 2 & 1 & T_3 & -15 \\ (5) & (2) & (-1) & -1 & 2 & 1 & T_4 & -3 \\ (-3) & (-1) & (2) & (3) & 1 & 3 & T_5 & 16 \\ (4) & (3) & (1) & (-6) & (-3) & -2 & T_6 & -27 \end{array}$$

$$\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & 4 \\ 2-(2/1) & -1-(2/1) & 1-(2/1)-2 & 2-(2/1) & 1-(2/1)3 & -3-(2/1)-1 & T_2 & \\ 1-(1/1) & 3-(1/1) & -3(1/1)-2 & -1(1/1) & 2-(1/1)3 & 1-(1/1)-1 & T_3 & \\ 5-(5/1) & 2-(5/1) & -1(5/1)-2 & -1(5/1) & 2-(5/1)3 & 1-(5/1)-1 & T_4 & \\ -3-(-3/1) & -1-(-3/1) & 2-(-3/1)-2 & 3-(-3/1) & 1-(-3/1)3 & 3-(-3/1)-1 & T_5 & \\ 4-(4/1) & 3-(4/1) & 1-(4/1)-2 & -6-(4/1) & -3-(4/1)3 & -2-(4/1) & T_6 & \end{array}$$

$$\begin{array}{cccccc|c|c} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & 4 \\ 0 & -2 & 5 & 0 & -5 & -1 & T_2 & 12 \\ 0 & (2) & -1 & -2 & -1 & 2 & T_3 & -19 \\ 0 & (-3) & (9) & -6 & -13 & 6 & T_4 & -23 \\ 0 & (2) & (-4) & (6) & (0) & 0 & T_5 & 28 \\ 0 & (1) & (9) & (-10) & (-15) & 2 & T_6 & 43 \end{array}$$

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 2 - (-\frac{2}{3}) \cdot 3 & -5 - (-\frac{2}{3}) \cdot 5 & -2 - (-\frac{2}{3}) \cdot 0 & -1 - (-\frac{2}{3}) \cdot (-5) & 2 - (-\frac{2}{3}) \cdot (-1) \\
 0 & -3 - (-\frac{2}{3}) \cdot 3 & 9 - (-\frac{2}{3}) \cdot 5 & -6 - (-\frac{2}{3}) \cdot 0 & -13 - (-\frac{2}{3}) \cdot (-5) & 6 - (-\frac{2}{3}) \cdot (-1) \\
 0 & 2 - (-\frac{2}{3}) \cdot 3 & -4 - (-\frac{2}{3}) \cdot 5 & 6 - (-\frac{2}{3}) \cdot 0 & 16 - (-\frac{2}{3}) \cdot (-5) & 0 - (-\frac{2}{3}) \cdot (-1) \\
 0 & -1 - (-\frac{1}{3}) \cdot 3 & 9 - (-\frac{1}{3}) \cdot 5 & -10 - (-\frac{1}{3}) & -15 - (-\frac{1}{3}) \cdot (-5) & 2 - (-\frac{1}{3}) \cdot (-1)
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}$$

$$2 \begin{bmatrix}
 4 \\
 12 \\
 -19 - (-\frac{2}{3}) \cdot 12 \\
 -23 - (-\frac{2}{3}) \cdot 12 \\
 28 - (-\frac{2}{3}) \cdot 12 \\
 -43 - (-\frac{1}{3}) \cdot 12
 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\
 0 & 0 & 4 & -6 & -8 & 7 \\
 0 & 0 & -0.7 & 6 & 7 & -0.7 \\
 0 & 0 & 7.4 & -10 & -13.4 & 2.4
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}
 \begin{bmatrix}
 4 \\
 12 \\
 -11 \\
 -35 \\
 36 \\
 -47
 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\
 0 & 0 & 4 - (\frac{1}{2.4}) \cdot 2.4 & -6 - (-\frac{1}{2.4}) \cdot 2 & -8 - (\frac{1}{2.4}) \cdot 4.4 & 7 - (\frac{1}{2.4}) \cdot 1.4 \\
 0 & 0 & -0.7 - (-\frac{0.7}{2.4}) \cdot 2.4 & 6 - (-\frac{0.7}{2.4}) \cdot 2 & 7 - (-\frac{0.7}{2.4}) \cdot 4.4 & -0.7 - (-\frac{0.7}{2.4}) \cdot 1.4 \\
 0 & 0 & 7.4 - (\frac{7.4}{2.4}) \cdot 2.4 & -10 - (\frac{7.4}{2.4}) \cdot 2 & -13.4 - (\frac{7.4}{2.4}) \cdot 4.4 & 2.4 - (\frac{7.4}{2.4}) \cdot 1.4
 \end{bmatrix}$$

$$\begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}
 \begin{bmatrix}
 4 \\
 12 \\
 -11 \\
 -35 - (\frac{1}{2.4}) \cdot (-11) \\
 36 - (-\frac{0.7}{2.4}) \cdot (-11) \\
 -47 - (\frac{7.4}{2.4}) \cdot (-11)
 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\
 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\
 0 & 0 & 0 & (5.42) & 5.42 & -0.09 \\
 0 & 0 & 0 & (-3.82) & (0.17) & -1.91
 \end{bmatrix}
 \begin{bmatrix}
 \tau_1 \\
 \tau_2 \\
 \tau_3 \\
 \tau_4 \\
 \tau_5 \\
 \tau_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 12 \\
 -11 \\
 -16.7 \\
 32.79 \\
 -13.08
 \end{bmatrix}$$

II

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\
 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\
 0 & 0 & 0 & 5.42 & -\left(\frac{5.42}{-2.7}\right) \cdot 0.7 & -2.7 \cdot 5.42 - \left(\frac{5.42}{-2.7}\right) \cdot 0.7 - 0.09 \cdot \left(\frac{5.42}{-2.7}\right) \\
 0 & 0 & 0 & -3.82 & -\left(\frac{3.82}{-2.7}\right) \cdot 0.7 & -2.7 \cdot (-3.82) - 0.17 \cdot \left(\frac{3.82}{-2.7}\right) - 1.91 \cdot \left(\frac{3.82}{-2.7}\right)
 \end{bmatrix}$$

$$\begin{bmatrix}
 \tau_1 \\
 \tau_2 \\
 \tau_3 \\
 \tau_4 \\
 \tau_5 \\
 \tau_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 12 \\
 -11 \\
 -16.7 \\
 32.79 - \left(\frac{5.42}{-2.7}\right)(-16.7) \\
 -13.08 - \left(\frac{-3.82}{-2.7}\right)(-16.7)
 \end{bmatrix}$$

III

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\
 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\
 0 & 0 & 0 & 0 & 4.01 & 7.2 \\
 0 & 0 & 0 & 0 & (1.16) & 8.59
 \end{bmatrix}
 \begin{bmatrix}
 \tau_1 \\
 \tau_2 \\
 \tau_3 \\
 \tau_4 \\
 \tau_5 \\
 \tau_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 12 \\
 -11 \\
 -16.7 \\
 -0.74 \\
 10.7
 \end{bmatrix}$$

IV

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 0 & -3 & 5 & 0 & -5 & -1 \\
 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\
 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\
 0 & 0 & 0 & 0 & 4.01 & 7.2 \\
 0 & 0 & 0 & 0 & 1.16 & -\left(\frac{1.16}{4.01}\right) \cdot 7.2 - 8.59 - \left(\frac{1.16}{4.01}\right) \cdot 7.2
 \end{bmatrix}
 \begin{bmatrix}
 \tau_1 \\
 \tau_2 \\
 \tau_3 \\
 \tau_4 \\
 \tau_5 \\
 \tau_6
 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ -0.76 \\ 10.7 - \left(\frac{11.36}{10.7} \right) (-0.76) \end{bmatrix}$$

The final matrix will be:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.7 & -2 & -4.4 & 1.4 \\ 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\ 0 & 0 & 0 & 0 & 4.01 & 9.2 \\ 0 & 0 & 0 & 0 & 0 & -11.3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ -0.76 \\ 10.92 \end{bmatrix}$$

From the above matrix, we can solve for the temperatures $T_1, T_2, T_3, T_4, T_5, T_6$ resp.

$$-11.36 T_6 = 10.92$$

$$T_6 = - \frac{10.92}{11.3}$$

$$T_6 = -0.966 \text{ --- (1)}$$

$$4.01 T_5 + 9.2 T_6 = -0.76$$

$$T_5 = \frac{-0.76 - 9.2(-0.966)}{4.01}$$

$$T_5 = \frac{8.1472}{4.01} = 2.031 \text{ --- (2)}$$

$$-2.7 T_4 - 0.7 T_5 + 4.7 T_6 = -16.7$$

$$-2.7 T_4 - 0.7(2.031) + 4.7(-0.966) = -16.7$$

$$-2.7 T_4 - 1.4217 - 4.5462 = -16.7$$

$$-2.7 T_4 = -16.7 + 1.4217 + 4.5462$$

$$-2.7 T_4 = -10.7381$$

$$T_4 = \frac{10.7381}{2.7} = 3.977 \text{ --- (3)}$$

$$2.4T_3 - 2T_4 - 4.4T_5 + 1.4T_6 = -11$$

$$2.4T_3 - 2(3.977) - 4.4(2.031) + 1.4(-0.966) = -11$$

$$2.4T_3 - 7.954 - 8.9364 - 1.3524 = -11$$

$$2.4T_3 = -11 + 18.2428$$

$$T_3 = \frac{7.2428}{2.4}$$

$$T_3 = 3.017 \dots \textcircled{4}$$

$$-3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$$

$$-3T_2 + 5(3.017) + 0 - 5(2.031) - 1(-0.966) = 12$$

$$-3T_2 + 15.085 - 10.155 + 0.966 = 12$$

$$-3T_2 = 6.104$$

$$T_2 = \frac{6.104}{3}$$

$$T_2 = -2.034 \dots \textcircled{5}$$

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - 3T_6 = 4$$

$$T_1 + (-2.034) - 2(3.017) + (3.977) + 3(2.031) + 0.966 = 4$$

$$T_1 - 2.034 - 6.034 + 3.977 + 6.093 + 0.966 = 4$$

$$T_1 = 1.032 \dots \textcircled{6}$$

From eqn ① to ⑥ resp.

$$T_1 = 1.032^\circ\text{C}$$

$$T_2 = -2.034^\circ\text{C}$$

$$T_3 = 3.017^\circ\text{C}$$

$$T_4 = 3.977^\circ\text{C}$$

$$T_5 = 2.031^\circ\text{C}$$

$$T_6 = -0.966^\circ\text{C}$$