

ADDEDIPE E. SEGUN

15/ENG006/001

MECHANICAL ENGINEERING,

ENG382

Using Gauss elimination method.

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$$

In matrix form

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & 1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

In Augmented matrix form

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & 0 & 0 & 0 & -2 & -27 \end{array} \right]$$

Row 2 - 2 × [Row 1].

$$2 - 2[1] = 0$$

$$-1 - 2[1] = -3$$

$$1 - 2[-2] = 5$$

$$2 - 2[1] = 0$$

$$1 - 2[3] = -5$$

$$-3 - 2[-1] = -1$$

$$20 - 2[4] = 12$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & 6 & -3 & -2 & -27 \end{array} \right]$$

$$\text{Row 3} - \frac{1}{1} [\text{Row 1}]$$

$$1 - 1[1] = 0$$

$$3 - 1[1] = 2$$

$$-3 - 1[-2] = -1$$

$$-1 - 1[1] = -2$$

$$2 - 1[3] = -1$$

$$1 - 1[-1] = 2$$

$$-15 - 1[4] = -19$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & 6 & -3 & 2 & -27 \end{array} \right]$$

$$\text{Row 3} - \frac{2}{-3} [\text{Row 2}]$$

$$0 - [-\frac{2}{3}] \times [0] = 0$$

$$2 - [-\frac{2}{3}] \times [-3] = 0$$

$$-1 - [-\frac{2}{3}] \times [5] = 2.33$$

$$-2 - [-\frac{2}{3}] \times [0] = -2$$

$$-1 - [-\frac{2}{3}] \times [-5] = -4.33$$

$$2 - [-\frac{2}{3}] \times [1] = 1.33$$

$$-19 - [-\frac{2}{3}] \times [12] = -11$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2/3 & -2 & -4/3 & 1/3 & -11 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & 27 \end{array} \right]$$

$$\text{Row 4} - 5/1 [\text{Row 1}]$$

$$5 - 5/1 \times 1 = 0$$

$$2 - 5/1 \times 1 = -3$$

$$-1 - 5/1 \times -2 = 9$$

$$-1 - 5/1 \times 1 = -6$$

$$2 - 5/1 \times 3 = -13$$

$$1 - 5/1 \times -1 = 6$$

$$-3 - 5/1 \times 4 = -23$$

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2/3 & -2 & -4/3 & 1/3 & -11 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ -3 & -1 & 2 & -3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & 27 \end{array} \right]$$

$$\text{Row 4} - [-3/3] \times \text{Row 2}$$

$$0 - 1 \times 0 = 0$$

$$-3 - 1 \times 0 = 0$$

$$9 - 1 \times 5 = 4$$

$$-6 - 1 \times 0 = -6$$

$$-13 - 1 \times -5 = -8$$

$$6 - 1 \times -1 = 7$$

$$-23 - 1 \times 12 = -35$$

Row 6 -  $(4/1)$  Row 1

$$4 - (4)1 = 0$$

$$3 - (4)1 = -1$$

$$1 - (4)(-2) = 9$$

$$-1 - (4)(1) = -5$$

$$-3 - (4)(3) = -15$$

$$-2 - (4)(-1) = 2$$

$$-27 - (4)(4) = -43$$

Row 5 -  $(-2/3)$  Row 2

$$2 - (-2/3)(-3) = 0$$

$$-4 - (-2/3)(5) = -2/3$$

$$6 - (-2/3)(0) = 6$$

$$10 - (-2/3)(5) = 20/3$$

$$0 - (-2/3)(-1) = -2/3$$

$$28 - (-2/3)(12) = 36$$

$$a_{62} = 6$$

Row 6 -  $(1/3)$  Row 2

$$a_{62} = -1 - (1/3)(-3) = 0$$

$$a_{63} = 9 - (1/3)(5) = 22/3$$

$$a_{64} = -5 - (1/3)(0) = -5$$

$$a_{65} = -15 - (1/3)(-5) = -40/3$$

$$a_{66} = 2 - (1/3)(-1) = 7/3$$

$$a_{67} = -43 - (1/3)(12) = -47$$

Augmented matrix

1	1	2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	0	7/3	-2	-13/3	4/3	-11
0	0	4	-6	-8	7	-35
0	0	-2/3	6	20/3	-2/3	36
0	0	22/3	-5	-40/3	7/3	-47

$$a_{42} = 0$$

Row 4 -  $(12/7)$  Row 3

$$a_{43} = 4 - (12/7)(7/3) = 0$$

$$a_{44} = -6 - (12/7)(-2) = -18/7$$

$$a_{45} = -8 - (12/7)(13/3) = -4/7$$

$$a_{46} = 7 - (12/7)(4/3) = 33/7$$

$$a_{47} = -35 - (12/7)(-11) = -113/7$$

To make  $a_{53} = 0$  [Row 5 -  $(-2/7)$  x Row 3]

$$a_{53} = -2/3 - (-2/7)(7/3) = 0$$

$$a_{54} = 6 - (-2/7)(-2) = 38/7$$

$$a_{55} = 20/3 - (-2/7)(13/3) = 38/7$$

$$a_{56} = -2/3 - (-2/7)(4/3) = -2/7$$

$$a_{57} = 36 - (-2/7)(-11) = 230/7$$

$$a_{63} = 0$$

Row 6 -  $(22/7)$  Row 3

$$a_{63} = 22/3 - (22/7)(7/3) = 0$$

$$a_{64} = -5 - (22/7)(-2) = 9/7$$

$$a_{65} = -40/3 - (22/7)(13/3) = 2/7$$

$$a_{66} = 7/3 - (22/7)(4/3) = -13/7$$

$$a_{67} = -47 - (22/7)(-11) = -87/7$$

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1 matrix formed

1	1	-2	1	3	-1	4
0	-3	5	0	-5	-1	12
0	0	7/3	-2	-13/3	4/3	-11
0	0	0	-18/7	-4/7	33/7	-113/7
0	0	0	38/7	38/7	-2/7	230/7
0	0	0	9/7	2/7	-13/7	-87/7

$$a_{54} = 0$$

$$\text{Row 6} - (a_{54}/a_{44})(-19/9) [\text{Row 4}]$$

$$a_{54} = 38/7 - (-19/9)(-18/7) = 0$$

$$a_{55} = 38/7 - (-19/9)(-4/7) = 38/9$$

$$a_{56} = -2/7 - (-19/9)(33/7) = 29/3$$

$$a_{57} = 230/7 - (-19/9)(-113/7) = -11/9$$

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$$\text{To make } a_{64} = 0$$

$$\text{Row 6} - (-1/2) \text{ Row 4}$$

$$a_{64} = 9/7 - (-1/2)(-18/7) = 0$$

$$a_{65} = 2/7 - (-1/2)(-4/7) = 0$$

$$a_{66} = -13/7 - (-1/2)(33/7) = 1/2$$

$$a_{67} = -87/7 - (-1/2)(-113/7) = -41/2$$

matrix formed

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -6 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & 113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & -41/2 \end{bmatrix}$$

$$a_{65} = 0$$

$$\text{Row 6} - (a_{65}/a_{55}) 10 \times [\text{Row 5}]$$

$$a_{65} = 0 - (0)(38/9) = 0$$

$$a_{66} = 1/2 - (0)(29/3) = 1/2$$

$$a_{67} = -41/2 - (0)(-11/9) = -41/2$$

matrix formed

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -18/7 & -4/7 & 33/7 & 113/7 \\ 0 & 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/2 & -41/2 \end{bmatrix}$$

Back substitution.

$$T_6 = 0T_1 + 0T_2 + 0T_3 + 0T_4 + 0T_5 + \frac{1}{2}[T_6] = -4\frac{1}{2}$$

using;

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & -5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & -\frac{33}{7} & \frac{33}{7} \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{29}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ -\frac{11}{9} \\ -4\frac{1}{2} \end{bmatrix}$$

$$\frac{T_6}{2} = \frac{-41}{2}$$

$$T_6 = -41$$

$$T_5 = 0T_1 + 0T_2 + 0T_3 + 0T_4 + \frac{38}{9}(T_5) + \frac{29}{3}(T_6) = -\frac{11}{9}$$

$$\frac{38T_5}{9} + \frac{29(-41)}{3} = -\frac{11}{9}$$

$$\frac{38T_5}{9} = \frac{3556}{9}$$

$$T_5 = 93.57895$$

$$T_4 = 0T_1 + 0T_2 + 0T_3 - \left(\frac{18}{7}\right)T_4 + \left(-\frac{4}{7}\right)T_5 + \frac{33}{7}T_6$$

$$-\frac{18}{7}T_4 - \frac{4}{7} \times \frac{3556}{38} + \frac{33}{7} \times (-41) = -\frac{113}{7}$$

$$\frac{18T_4}{7} = \frac{30672}{133}$$

$$T_4 = -89.6842 \approx -89.7$$

$$T_3 = 0T_1 + 0T_2 + \left(\frac{7}{3}\right)T_3 + (-2)T_4 + \left(-\frac{13}{3}\right)T_5 + \left(\frac{4}{3}\right)T_6$$

$$\frac{7}{3}T_3 - \left(2 \times -\frac{1704}{19}\right) - \left(\frac{13}{3} \times \frac{3556}{38}\right) + \left(\frac{4}{3} \times -\frac{41}{1}\right)$$

$$= -11$$

$$\frac{7}{3}T_3 = -11 - \frac{3408}{17} + \frac{46228}{114} + \frac{164}{3}$$



$$\frac{7}{3} T_3 = \frac{-18379}{57}$$

$$T_3 = \frac{15379}{57} - \frac{7}{3}$$

$$T_3 = 115.63$$

$$T_2 = 0T_1 - 3T_2 + 5T_3 + 0T_4 - 5T_5 - T_6 = 12$$

$$-3T_2 + 5T_3 - 5T_5 - T_6 = 12$$

$$-3T_2 + \left(5 \times \frac{46137}{399}\right) - 5 \times 93.57 - (-41) = 12$$

$$-3T_2 = -578.15 - 426.85 + 12$$

$$T_2 = 46.42$$

$$T_1 = T_1 + T_2 + (-2)T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 + \frac{882}{19} + (-2 \times \frac{46137}{399}) + (-\frac{1704}{19}) + (3 \times \frac{3556}{38})$$

$$+ (-1 \times -41) = 4$$

$$T_1 = 4 - 41 - \frac{882}{19} + \frac{92274}{399} + \frac{1708}{19} - \frac{10668}{38}$$

$$T_1 = \underline{\underline{-43.21}}$$