ASSIGNMENT ONE

NAME: NWOSU CHISOM JOY.

MATRIC NO: U/ENG01/021.

COURSE CODE: CHE 532

COURSE TITLE: PROCESS DYNAMICS AND CONTROL II

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From the question,
A Constant volume isothermal reactor in which a first order reaction given as $A \xrightarrow{k} B$ takes place.

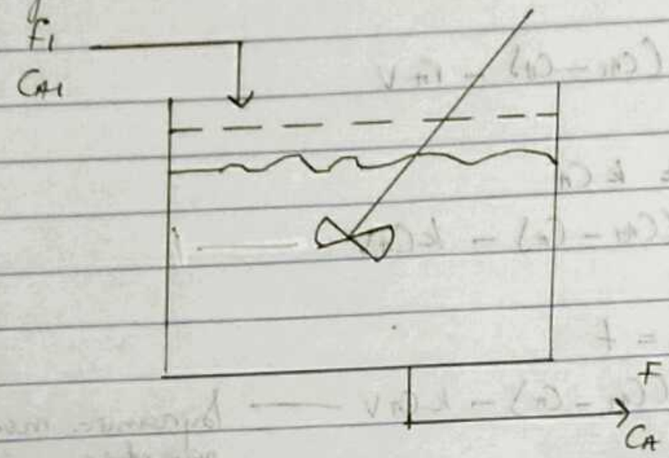

Figure 1: A constant Volume Isothermal Continuous Stirred tank Reactor.

Recall from first Semester 500level (Process Dynamics and Control I);

Mass balance

$$\frac{dv}{dt} = f_i - f$$

Mole balance

$$\frac{dn}{dt} = n_i - n \pm rv \quad (\text{Accumulation} = \text{in} - \text{Out} \pm \text{Conversion})$$

but $n = cv$

$$\frac{dcv}{dt} = C_i f_i - Cf \pm rv$$

Considering component A

$$\frac{d(C_A V)}{dt} = C_{A_i} f_i - C_A f - r_A V$$

Using product rule

$$V\frac{dC_A}{dt} + C_A\frac{dv}{dt} = C_{Ai}f_i - C_Af - r_AV$$

$$V\frac{dC_A}{dt} = C_{Ai}f_i - C_Af - r_AV - C_A\frac{dv}{dt}$$

$$V\frac{dC_A}{dt} = C_{Ai}f_i - C_Af - r_AV - C_A(f_i - f)$$ [from $dv/dt = f_i - f$]

$$V\frac{dC_A}{dt} = C_{Ai}f_i - \cancel{C_Af} - r_AV - C_Af_i + \cancel{C_Af}$$

$$V\frac{dC_A}{dt} = C_{Ai}f_i - C_Af_i - r_AV$$

$$V\frac{dC_A}{dt} = f_i(C_{Ai} - C_A) - r_AV$$

but $r_A = kC_A$

$$V\frac{dC_A}{dt} = f_i(C_{Ai} - C_A) - kC_AV$$

but $f_i = f$

$$V\frac{dC_A}{dt} = f(C_{Ai} - C_A) - kC_AV$$ —— Dynamic model of the reactor

$$V\frac{dC_{As}}{dt} = f(C_{Ais} - C_{As}) - kC_{As}V$$ —— Steady state model of the reactor

Dynamic model − Steady state model

$$V\frac{d(C_A - C_{As})}{dt} = f(C_{Ai} - C_A) - f(C_{Ais} - C_{As}) - kV(C_A - C_{As})$$

$$V\frac{d\bar{C}_A}{dt} = f\bar{C}_{Ai} - f\bar{C}_A - kV\bar{C}_A$$

Taking the Laplace of both sides

$$V[s\bar{C}_A(s) - \bar{C}_A(0)] = f\bar{C}_{Ai}(s) - f\bar{C}_A(s) - kV\bar{C}_A(s)$$ [from $f'(t) = sf(s) - f(0)$]

$$Vs\bar{C}_A(s) + \bar{f}C_A(s) + k\bar{V}C_A(s) = f\bar{C}_{Ai}(s)$$

$$f\bar{C}_{Ai}(s) = Vs\bar{C}_A(s) + \bar{f}C_A(s) + k\bar{V}C_A(s)$$

Collecting like terms

$$\bar{C}_A(s)[Vs + f + kV] = f\bar{C}_{Ai}(s)$$

Recall $G(s) = \frac{\text{Output}}{\text{Input}} = \frac{\bar{C}_A(s)}{\bar{C}_{Ai}(s)}$

$$G(s) = \frac{f}{Vs + f + kV}$$

∴ The transfer function of the above system $G(s) = \frac{f}{Vs + f + kV}$