

Akomolafe David Olusegun

15/ENG01/003

CHEMICAL ENGINEERING

ENG 382

If the model of a system having thermocouples measuring temperatures $T(^{\circ}\text{C})$ at its different points is given by the set of expressions in equation 1, estimate the values of the temperatures using

- i. Gauss elimination method
- ii. Gauss elimination method with the aid of excel
- iii. Matrix inverse method with aid of excel
- iv. Matrix inverse method with the aid of MATLAB

$$\left\{ \begin{array}{l} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{array} \right\} \quad \text{--- (1)}$$

Solution

$$\begin{array}{cccccc} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & 1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{array} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

$$\begin{array}{ccccccc}
 1 & 1 & -2 & 1 & 3 & -1 & -4 \\
 \left(2 - \left(\frac{2}{1}\right) \times 1\right) & \left(-1 - \left(\frac{2}{1}\right) \times 1\right) & \left(1 - \left(\frac{2}{1}\right) \times 2\right) & \left(2 - \left(\frac{2}{1}\right) \times 1\right) & \left(1 - \left(\frac{2}{1}\right) \times 3\right) & \left(-3 - \left(\frac{2}{1}\right) \times 1\right) & 20 - \left(\frac{2}{1}\right) \times 4 \\
 \left(1 - \left(\frac{1}{1}\right) \times 1\right) & \left(3 - \left(\frac{1}{1}\right) \times 1\right) & \left(-3 - \left(\frac{1}{1}\right) \times 2\right) & \left(-1 - \left(\frac{1}{1}\right) \times 1\right) & \left(2 - \left(\frac{1}{1}\right) \times 3\right) & \left(1 - \left(\frac{1}{1}\right) \times 1\right) & 15 - \left(\frac{1}{1}\right) \times 4 \\
 \left(5 - \left(\frac{5}{1}\right) \times 1\right) & \left(2 - \left(\frac{5}{1}\right) \times 1\right) & \left(-1 - \left(\frac{5}{1}\right) \times 2\right) & \left(-1 - \left(\frac{5}{1}\right) \times 1\right) & \left(2 - \left(\frac{5}{1}\right) \times 3\right) & \left(1 - \left(\frac{5}{1}\right) \times 1\right) & -3 - \left(\frac{5}{1}\right) \times 4 \\
 \left(-3 - \left(\frac{-3}{1}\right) \times 1\right) & \left(-1 - \left(\frac{-3}{1}\right) \times 1\right) & \left(2 - \left(\frac{-3}{1}\right) \times 2\right) & \left(3 - \left(\frac{-3}{1}\right) \times 1\right) & \left(1 - \left(\frac{-3}{1}\right) \times 3\right) & \left(3 - \left(\frac{-3}{1}\right) \times 1\right) & 16 - \left(\frac{-3}{1}\right) \times 4 \\
 \left(4 - \left(\frac{4}{1}\right) \times 1\right) & \left(3 - \left(\frac{4}{1}\right) \times 1\right) & \left(1 - \left(\frac{4}{1}\right) \times 2\right) & \left(-6 - \left(\frac{4}{1}\right) \times 1\right) & \left(-3 - \left(\frac{4}{1}\right) \times 3\right) & \left(-2 - \left(\frac{4}{1}\right) \times 1\right) & -27 - \left(\frac{4}{1}\right) \times 4
 \end{array}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & -43 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & \left(2 - \left(\frac{2}{3}\right) \times 3\right) & \left(-1 - \left(\frac{2}{3}\right) \times 5\right) & \left(-2 - \left(\frac{2}{3}\right) \times 1\right) & \left(-1 - \left(\frac{2}{3}\right) \times 5\right) & \left(2 - \left(\frac{2}{3}\right) \times 1\right) & -19 - \left(\frac{2}{3}\right) \times 12 \\ 0 & \left(-3 - \left(\frac{-3}{3}\right) \times 3\right) & \left(9 - \left(\frac{-3}{3}\right) \times 5\right) & \left(-6 - \left(\frac{-3}{3}\right) \times 1\right) & \left(-13 - \left(\frac{-3}{3}\right) \times 5\right) & \left(6 - \left(\frac{-3}{3}\right) \times 1\right) & -23 - \left(\frac{-3}{3}\right) \times 12 \\ 0 & \left(2 - \left(\frac{2}{3}\right) \times 3\right) & \left(-4 - \left(\frac{2}{3}\right) \times 5\right) & \left(6 - \left(\frac{2}{3}\right) \times 1\right) & \left(10 - \left(\frac{2}{3}\right) \times 5\right) & \left(0 - \left(\frac{2}{3}\right) \times 1\right) & 28 - \left(\frac{2}{3}\right) \times 12 \\ 0 & \left(-1 - \left(\frac{-1}{3}\right) \times 3\right) & \left(9 - \left(\frac{-1}{3}\right) \times 5\right) & \left(-10 - \left(\frac{-1}{3}\right) \times 1\right) & \left(-15 - \left(\frac{-1}{3}\right) \times 5\right) & \left(2 - \left(\frac{-1}{3}\right) \times 1\right) & -43 - \left(\frac{-1}{3}\right) \times 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & -35 \\ 0 & 0 & -0.67 & 6 & 6.667 & -0.67 & 36 \\ 0 & 0 & 7.33 & 10 & -13.33 & 2.33 & -47 \\ 0 & & & & & & \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & 1 & -11 \\ 0 & 0 & \left(4 - \left(\frac{4}{2.33}\right) \times 2.33\right) & \left(-6 - \left(\frac{4}{2.33}\right) \times -2\right) & \left(-6 - \left(\frac{4}{2.33}\right) \times -4.33\right) & \left(2 - \left(\frac{4}{2.33}\right) \times 1.33\right) & 1 & -35 - \left(\frac{4}{2.33}\right) \times 11 \\ 0 & 0 & \left(-0.67 - \left(\frac{-0.67}{2.33}\right) \times 2.33\right) & \left(6 - \left(\frac{-0.67}{2.33}\right) \times -2\right) & \left(6 - \left(\frac{-0.67}{2.33}\right) \times -4.33\right) & \left(-0.67 - \left(\frac{-0.67}{2.33}\right) \times 1.33\right) & 1 & 56 - \left(\frac{-0.67}{2.33}\right) \times 11 \\ 0 & 0 & \left(7.33 - \left(\frac{7.33}{2.33}\right) \times 2.33\right) & \left(-10 - \left(\frac{7.33}{2.33}\right) \times -2\right) & \left(-10 - \left(\frac{7.33}{2.33}\right) \times -4.33\right) & \left(2.33 - \left(\frac{7.33}{2.33}\right) \times 1.33\right) & 1 & -47 - \left(\frac{7.33}{2.33}\right) \times 11 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & 1 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & 1 & -16.14 \\ 0 & 0 & 0 & 5.42 & 5.42 & -0.28 & 1 & 32.85 \\ 0 & 0 & 0 & -3.71 & 0.28 & -1.85 & 1 & -12.43 \end{bmatrix}$$

$$a_{54} = 5.42 - \left(\frac{-5.42}{2.57}\right) \times 2.57 = 0$$

$$a_{55} = 5.42 - \left(\frac{-5.42}{2.57}\right) \times -0.57 = 4.22$$

$$a_{56} = -0.28 - \left(\frac{-5.42}{2.57}\right) \times 4.71 = 9.67$$

$$b_5 = 32.85 - \left(\frac{-5.42}{2.57}\right) \times -16.14 = -1.22$$

$$a_{64} = -3.71 - \left(\frac{-3.71}{2.57}\right) \times 2.57 = 0$$

$$a_{65} = 0.28 - \left(\frac{-3.71}{2.57}\right) \times -0.57 = 1.11$$

$$a_{66} = -1.85 - \left(\frac{-3.71}{2.57}\right) \times 4.71 = -8.67$$

$$b_6 = -12.43 - \left(\frac{-3.71}{2.57}\right) \times -16.14 = 10.89$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & 1 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & 1 & -16.14 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 & 1 & -1.22 \\ 0 & 0 & 0 & 0 & 1.11 & -8.67 & 1 & 10.89 \end{bmatrix}$$

$$a_{65} = 1.11 - \left(\frac{1.11}{4.22}\right) \cdot 4.22 = 0$$

$$b_6 = 10.89 - \left(\frac{10.11}{4.22}\right) \cdot 4.22 = 11.21$$

$$a_{66} = -8.67 - \left(\frac{-8.67}{4.22}\right) \cdot 9.67 = -11.21$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 & -11 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 & -16.14 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 & -1.22 \\ 0 & 0 & 0 & 0 & 0 & -11.21 & 11.21 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.33 & -2 & -4.33 & 1.33 \\ 0 & 0 & 0 & -2.57 & -0.57 & 4.71 \\ 0 & 0 & 0 & 0 & 4.22 & 9.67 \\ 0 & 0 & 0 & 0 & 0 & -11.21 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.14 \\ -1.22 \\ 11.21 \end{bmatrix}$$

$$-11.21 \cdot T_6 = 11.21$$

$$T_6 = -1$$

$$ii) \quad 4.22 T_5 + 9.67 T_6 = -1.22$$

$$4.22 T_5 + 9.67(-1) = -1.22$$

$$T_5 = \frac{-1.22 + 9.67}{4.22}$$

$$= 2$$

$$iii) \quad -2.57 T_4 = -16.14 + 0.57(2) + 4.71(-1)$$

$$T_4 = \frac{-16.14 + 0.57(2) + 4.71}{-2.57}$$

$$= 4$$

$$T_4 = 4$$

$$\text{IV } 2.33T_3 - 2T_4 - 4.33T_5 + 1.33T_6 = -11$$

$$2.33T_3 = -11 + 2T_4 + 4.33T_5 - 1.33T_6$$

$$T_3 = \frac{-11 + 2(4) + 4.33(2) - 1.33(-1)}{2.33}$$

$$T_3 = 3$$

$$\text{V } -3T_2 + 5T_3 + 10T_4 - 5T_5 - 17T_6 = 12$$

$$T_2 = \frac{12 - 5T_3 - 10T_4 - 5T_5 - 17T_6}{-3}$$

$$T_2 = \frac{12 - 5(3) - 10(2) - 5(-1) - 17(-1)}{-3}$$

$$T_2 = -2$$

$$= T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 = 4 - T_2 - 2T_3 - T_4 - 3T_5 + T_6$$

$$T_1 = 4 - (-2) - 2(3) - 4 - 3(2) + (-1)$$

$$= 4 + 2 + 6 - 4 - 6 - 1$$

$$= 1$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \\ 2 \\ 1 \end{bmatrix}$$

$$\therefore T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = 1$$