

ENG 382 ASSIGNMENT 3

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 5T_5 - 2T_6 = -27$$

Transferring the equations to matrix

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -5 & -2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{pmatrix}$$

Using a factor of 2 to multiply Row 1 and subtract Row 2

$$2 - (2)(1); -1 - (2)(1); 1 - (2)(-2); 2 - (2)(1); 1 - (2)(3); -3 - (2)(-1); 20 - (2)(4)$$

$$0 \quad -3 \quad 5 \quad 0 \quad -5 \quad 1 \quad 12$$

Using a factor 1 to multiply Row 1 and subtract Row 3

$$1 - (1)(1); 3 - (1)(1); -3 - (1)(-2); -1 - (1)(1); 2 - (1)(3); 1 - (1)(-1); -15 - (1)(4)$$

$$0 \quad 2 \quad -1 \quad 2 \quad -1 \quad 2 \quad -19$$

Using a factor 5 to multiply Row 5 and subtract Row 4

$$5 - (5)(1); 2 - (5)(1); -1 - (5)(-2); -1 - (5)(1); 2 - (5)(3); 1 - (5)(-1); -3 - (5)(4)$$

$$0 \quad -3 \quad 9 \quad -6 \quad -13 \quad 6 \quad -23$$

Using a factor (3) to multiply Row 1 and subtract Row 5

$$-3 - (3)(1); -1 - (3)(1); 2 - (3)(-2); 3 - (3)(1); 1 - (3)(3); 3 - (3)(-1); 16 - (3)(4)$$

$$0 \quad 2 \quad -4 \quad 6 \quad 10 \quad 0 \quad 28$$

Using a factor 4 to multiply Row 1 and subtract Row 6

$$4 - (4)(1); 3 - (4)(1); 1 - (4)(-2); -6 - (4)(1); -3 - (4)(3); -2 - (4)(-1); -27 - (4)(4)$$

$$0 \quad -1 \quad 9 \quad -10 \quad -15 \quad 2 \quad -43$$

The matrix becomes:

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & 2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & -10 & -15 & 2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{pmatrix}$$

Using a factor of $(-2/3)$ to multiply Row 2 and subtract from Row 3

$$-(-2/3)(3); -1 - (-2/3)(5); -2 - (-2/3)(6); -1 - (-2/3)(-5); 2 - (-2/3)(-1); -19 - (-2/3)(12)$$

$$0 \quad 2.3333 \quad -2 \quad -4.3333 \quad 1.3333 \quad -11$$

Using a factor of 1 to multiply Row 2 and subtract Row 4

$$-3 - (1)(-3); 9 - (1)(5); -6 - (1)(6); -13 - (1)(-5); 6 - (1)(-1); 23 - (1)(12)$$

$$0 \quad 4 \quad -6 \quad -8 \quad 7 \quad -35$$

Using a factor of $(-2/3)$ to multiply Row 2 and subtract Row 5

$$-(-2/3)(-3); -4 - (-2/3)(5); 6 - (-2/3)(6); 10 - (-2/3)(-5); 0 - (-2/3)(-1); 28 - (-2/3)(12)$$

$$0 \quad -0.6666 \quad 6 \quad 6.6666 \quad 0.6666 \quad 36$$

Using a factor of $(1/3)$ to multiply Row 2 and subtract Row 6

$$-1 - (1/3)(-3); 9 - (1/3)(5); -10 - (1/3)(6); -15 - (1/3)(-5); 2 - (1/3)(-1); -43 - (1/3)(12)$$

$$0 \quad 7.3333 \quad -16 \quad -15.3333 \quad 2.3333 \quad -47$$

The row matrix becomes:

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 6 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -0.6666 & 6 & 6.6666 & 0.6666 \\ 0 & 0 & 7.3333 & -16 & -13.3333 & 2.3333 \end{pmatrix} = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix}$$

Using a factor of 0.2856 to multiply Row 3 and subtract Row 5

$$-0.6666(-0.2856)(2.3333); 6 - (-0.2856)(-2); 6.666 - (-0.2856)(-4.3333)$$

$$0 \quad 5.4285 \quad 5.4285$$

$$-0.6666 - (-0.2856)(1.3333); 36 - (-0.2856)(-11)$$

$$-0.2857 \quad 32.85714$$

Using a factor of $(7.3333/2.3333)$ to multiply Row 2 and subtract Row 6

$$7.3333 - (3.1428)(2.3333); -10 - (3.1428)(-2); -13.3333 - (3.1428)(-4.3333)$$

$$0 \quad -3.71429 \quad 0.2857$$

$$2.3333 - (3.1428)(1.3333); -41 - (3.1428)(-11)$$

$$-1.85714 \quad -12.4286$$

The new matrix becomes

$$\left(\begin{array}{cccccc|ccc} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & T_2 & & 12 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 & T_3 & = & -11 \\ 0 & 0 & 0 & -2.571 & -0.5714 & 4.7142 & T_4 & & -16.1429 \\ 0 & 0 & 0 & 5.4285 & 5.4285 & -0.2857 & T_5 & & 32.8571 \\ 0 & 0 & 0 & -3.7142 & 0.2857 & -1.8571 & T_6 & & -12.4286 \end{array} \right)$$

Using a factor of $(5.4285 / -2.571)$ to multiply Row 4 and subtract from Row 5

$$5.4285 - (-2.1114)(-2.571); 5.4285 - (-2.1114)(-0.5714); -0.2857 - (-2.1114)(4.7142);$$

$$0 \qquad 4.2222 \qquad 9.6666$$

$$32.8571 - (-2.1114)(-16.1429)$$

$$= 1.2222$$

Using a factor of $(-3.7142 / -2.571)$ to multiply Row 5 and divide Row 6

$$-3.7142 - (1.4446)(-2.571); 0.2857 - (1.4446)(-0.5714); -1.8571 - (1.4446)(4.7142);$$

$$0 \qquad 1.1111 \qquad -8.6666$$

$$-12.4286 - (1.4446)(-16.1429)$$

$$10.8888$$

The new matrix becomes

$$\left(\begin{array}{cccccc|ccc} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & T_2 & & 12 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 & T_3 & = & -11 \\ 0 & 0 & 0 & -2.5714 & -0.5714 & 4.7142 & T_4 & & -16.1429 \\ 0 & 0 & 0 & 0 & 4.2222 & 9.6666 & T_5 & & -1.2222 \\ 0 & 0 & 0 & 0 & 1.1111 & -8.6666 & T_6 & & 10.8888 \end{array} \right)$$

Using a factor of $(1.1111 / 4.2222)$ to multiply Row 5 and subtract Row 6

$$1.1111 - (0.2631)(4.2222); -8.6666 - (0.2631)(9.6666); 10.8888 - (0.2631)(-1.2222)$$

$$0 \qquad -11.2105 \qquad 11.2105$$

The new matrix becomes

$$\left(\begin{array}{cccccc|ccc} 1 & 1 & -2 & 1 & 3 & -1 & T_1 & & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & T_2 & & 12 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 & T_3 & = & -11 \\ 0 & 0 & 0 & -2.5714 & -0.5714 & 4.7142 & T_4 & & -16.1429 \\ 0 & 0 & 0 & 0 & 4.222 & 9.6666 & T_5 & & -1.2222 \\ 0 & 0 & 0 & 0 & 0 & -11.2105 & T_6 & & 11.2105 \end{array} \right)$$

Using back substitution

$$-11.2105 T_6 = 11.2105$$

$$T_6 = \frac{11.2105}{-11.2105} = -1$$

$$4.2222 T_5 + 9.6666 T_6 = -1.2222$$

$$T_5 = \frac{-1.2222 - 9.6666(-1)}{4.2222} = 2$$

$$-2.5714 T_4 - 0.5714 T_5 + 4.7143 T_6 = -16.1429$$

$$T_4 = \frac{-16.1429 - 4.7143(-1) + 0.5714 \times 2}{-2.5714} = 4$$

$$2.3333 T_3 - 2 T_4 + 4.3333 T_5 + 1.3333 T_6 = -11$$

$$T_3 = \frac{-11 - 1.3333(-1) + 4.3333(2) + 2(4)}{-2.3333} = 3$$

$$\Rightarrow -3 T_2 + 5 T_3 + 6 T_4 - 5 T_5 - T_6 = 12$$

$$T_2 = \frac{12 + (-1) + 5(2) - 5(3) - 6(4)}{-3} = -2$$

$$\Rightarrow T_1 + T_2 - 2 T_3 + T_4 + 3 T_5 - T_6 = 4$$

$$T_1 = \frac{4 + (-1) - 3(2) - 4 + 2(3) - (-2)}{1} = 1$$

Therefore

$$T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$