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1 i)

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$$

solution

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{array} \right]$$

$$a_{21} \quad 0 \quad \text{Row 2} - \frac{2}{1} \text{Row 1} \\ = \text{Row 2} - 2\text{Row 1}$$

$$\text{i) } 2 - 2(1) = 0$$

$$\text{ii) } -1 - 2(1) = -3$$

$$\text{iii) } 1 - 2(-2) = 5$$

$$\text{iv) } 2 - 2(1) = 0$$

$$\text{v) } 1 - 2(3) = -5$$

$$\text{vi) } -3 - 2(1) = -1$$

$$\text{vii) } 20 - 2(4) = 12$$

$$\text{Row 2} \rightarrow [0 \quad -3 \quad 5 \quad 0 \quad -5 \quad -1 \quad 12]$$

$$a_{31} \quad 0 \quad \text{Row 3} - \frac{5}{1} \text{Row 1} \\ = \text{Row 3} - 5\text{Row 1}$$

- i) $1 - 1(C_1) = 0$
- ii) $3 - 1(C_1) = 2$
- iii) $-3 - 1(C_2) = -1$
- iv) $-1 - 1(C_1) = -2$
- v) $2 - 1(C_3) = -1$
- vi) $1 - 1(C_4) = 2$
- vii) $-15 - 1(C_4) = -19$

Row 3 $\rightarrow [0 \quad 2 \quad -1 \quad -2 \quad -1 \quad 2 \quad -19]$

$q_4, 0$ Row 4 $= \frac{5}{1}$ Row 1

- i) $5 - 5(C_1) = 0$
- ii) $2 - 5(C_1) = -3$
- iii) $-1 - 5(C_2) = 9$
- iv) $-1 - 5(C_1) = -6$
- v) $2 - 5(C_3) = -13$
- vi) $1 - 5(C_4) = 6$
- vii) $-3 - 5(C_4) = -23$

Row 4 $\rightarrow [0 \quad -3 \quad 9 \quad -6 \quad -13 \quad 6 \quad -23]$

$q_5, 0$ Row 5 $= \frac{-3}{1}$ (Row 1)

Row 5 $+ 3$ Row 1

- i) $-3 + 3(C_1) = 0$
- ii) $-1 + 3(C_1) = 2$
- iii) $2 + 3(C_2) = -4$
- iv) $3 + 3(C_1) = 6$
- v) $1 + 3(C_3) = 10$
- vi) $3 + 3(C_4) = 0$
- vii) $16 + 3(C_4) = 28$

Row 5 $\rightarrow [0 \quad 2 \quad -4 \quad 6 \quad 10 \quad 0 \quad 28]$

$q_6, 0$ Row 6 $= \frac{4}{1}$ Row 1

- i) $4 - 4(C_1) = 0$
- ii) $3 - 4(C_1) = -1$
- iii) $1 - 4(C_2) = 9$
- iv) $-6 - 4(C_1) = -10$

$$v) -3 - 4(3) = -15$$

$$vi) -2 - 4(-1) = 2$$

$$vii) -27 - 4(4) = -43$$

$$\text{Row 6} \rightarrow [0 \ -1 \ 9 \ -10 \ -15 \ 2 \ -43]$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & -10 & -15 & 2 & -43 \end{bmatrix}$$

$$a_{32} \ 0 \quad \text{Row 3} - \frac{2}{-3} \text{Row 2}$$

$$i) 2 - \frac{2}{-3}(-3) = 0$$

$$ii) -1 - \frac{2}{-3}(5) = \frac{7}{3}$$

$$iii) -2 - \left(\frac{2}{-3}\right)(0) = -2$$

$$iv) -1 - \left(\frac{2}{-3}\right)(-5) = \frac{-13}{3}$$

$$v) 2 - \left(\frac{2}{-3}\right)(-1) = \frac{4}{3}$$

$$vi) -19 - \left(\frac{2}{-3}\right)(12) = -11$$

$$\text{new Row 3} \rightarrow [0 \ 0 \ \frac{7}{3} \ -2 \ \frac{-13}{3} \ \frac{4}{3} \ -11]$$

$$a_{42} \ 0 \quad \text{Row 4} - \left(\frac{-3}{-3}\right) \text{Row 2} \\ = \text{Row 4} - \text{Row 2}$$

$$i) -3 - 1(-3) = 0$$

$$ii) 9 - 1(-5) = 14$$

$$iii) -6 - 1(0) = -6$$

$$iv) -13 - 1(-5) = -8$$

$$v) 6 - 1(-1) = 7$$

$$vi) -23 - 1(12) = -35$$

$$\text{new Row 4} \rightarrow [0 \ 0 \ \frac{7}{3} \ 4 \ \frac{20}{3} \ \frac{20}{3} \ -35]$$

$$\text{new Row 4} \rightarrow [0 \ 0 \ 4 \ -6 \ -8 \ 7 \ -35]$$

$$a_{52} \ 0 \quad \text{Row 5} - \frac{2}{-3} \text{Row 2}$$

$$= \text{Row 5} + \frac{2}{3} \text{Row 2}$$

$$i) 2 + \frac{2}{3}(-3) = 0$$

$$ii) -4 + \frac{2}{3}(5) = \frac{-2}{3}$$

$$iii) 6 + \frac{2}{3}(0) = 6$$

$$iv) 10 + \frac{2}{3}(-5) = \frac{20}{3}$$

$$v) 0 + \frac{2}{3}(-1) = \frac{-2}{3}$$

$$vi) 28 + \frac{2}{3}(12) = 36$$

$$\text{New Row 5} \rightarrow \left[0 \quad 0 \quad \frac{-2}{3} \quad 6 \quad \frac{20}{3} \quad \frac{-2}{3} : 36 \right]$$

$$a_{c2} \quad 0 \quad \text{Row 6} - \frac{1}{3} \text{Row 2}$$

$$i) -1 - \frac{1}{3}(-3) = 0$$

$$ii) -9 - \frac{1}{3}(5) = \frac{22}{3}$$

$$iii) -10 - \frac{1}{3}(0) = -10$$

$$iv) -15 - \frac{1}{3}(-5) = \frac{-40}{3}$$

$$v) 2 - \frac{1}{3}(-1) = \frac{2}{3}$$

$$vi) -43 - \frac{1}{3}(12) = -47$$

$$\text{New Row 6} \rightarrow \left[0 \quad 0 \quad \frac{22}{3} \quad -10 \quad \frac{-40}{3} \quad \frac{7}{3} : -47 \right]$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & -5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & \frac{-13}{3} & \frac{4}{3} & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & -35 \\ 0 & 0 & \frac{-2}{3} & 6 & \frac{20}{3} & \frac{-2}{3} & 36 \\ 0 & 0 & \frac{22}{3} & -10 & \frac{-40}{3} & \frac{7}{3} & -47 \end{bmatrix}$$

$$a_{43} \quad 0 \quad \text{Row 4} - \left(\frac{4}{\frac{7}{3}} \right) \text{Row 3}$$

$$\text{Row 4} - \frac{12}{7} \text{Row 3}$$

$$i) 4 - \frac{12}{7} \left(\frac{7}{3} \right) = 0$$

$$ii) -6 - \left(\frac{12}{7} \right) (-2) = \frac{-18}{7}$$

$$iii) -8 - \left(\frac{12}{7} \right) \left(\frac{-13}{3} \right) = \frac{-4}{7}$$

$$iv) 7 - \frac{12}{7} \left(\frac{4}{3} \right) = \frac{38}{7}$$

$$v) -35 - \frac{12}{7} (-11) = \frac{-113}{7}$$

$$\text{Row 4} \rightarrow \left[0 \quad 0 \quad 0 \quad \frac{-18}{7} \quad \frac{-4}{7} \quad \frac{33}{7} : \frac{-113}{7} \right]$$

$$953 \quad 0 \quad \text{Row 5} - \left(\frac{-2}{3} \div \frac{7}{3} \right) \text{Row 3} = \text{Row 5} + \frac{2}{7} \text{Row 3}$$

$$i) \quad \frac{-2}{3} + \frac{2}{7} \left(\frac{7}{3} \right) = 0$$

$$ii) \quad 0 + \frac{2}{7} (-2) = \frac{-38}{7}$$

$$iii) \quad \frac{20}{3} + \frac{2}{7} \left(\frac{-13}{3} \right) = \frac{38}{7}$$

$$iv) \quad \frac{-2}{3} + \frac{2}{7} \left(\frac{4}{3} \right) = \frac{-2}{7}$$

$$v) \quad 30 + \frac{2}{7} (-11) = \frac{230}{7}$$

$$\text{new Row 5} \rightarrow \left[0 \quad 0 \quad 0 \quad \frac{38}{7} \quad \frac{38}{7} \quad \frac{-2}{7} \quad : \quad \frac{230}{7} \right]$$

$$963 \quad 0 \quad \text{Row 6} - \left(\frac{22}{3} \div \frac{7}{3} \right) \text{Row 3} = \text{Row 6} - \frac{22}{7} \text{Row 3}$$

$$i) \quad \frac{22}{3} - \frac{22}{7} \left(\frac{7}{3} \right) = 0$$

$$ii) \quad -10 - \frac{22}{7} (-2) = \frac{-26}{7}$$

$$iii) \quad \frac{-40}{3} - \frac{22}{7} \left(\frac{-13}{3} \right) = \frac{2}{7}$$

$$iv) \quad \frac{7}{3} - \frac{22}{7} \left(\frac{4}{3} \right) = \frac{-13}{7}$$

$$v) \quad -47 - \frac{22}{7} (-11) = \frac{-87}{7}$$

$$\text{new Row 6} \rightarrow \left[0 \quad 0 \quad 0 \quad \frac{-26}{7} \quad \frac{2}{7} \quad \frac{-13}{7} \quad : \quad \frac{-87}{7} \right]$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & | & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & | & 12 \\ 0 & 0 & \frac{7}{3} & -2 & \frac{-13}{3} & \frac{4}{3} & | & -11 \\ 0 & 0 & 0 & \frac{-18}{7} & \frac{-4}{7} & \frac{32}{7} & | & \frac{-113}{7} \\ 0 & 0 & 0 & \frac{38}{7} & \frac{38}{7} & \frac{-2}{7} & | & \frac{230}{7} \\ 0 & 0 & 0 & \frac{-26}{7} & \frac{2}{7} & \frac{-13}{7} & | & \frac{-87}{7} \end{bmatrix}$$

$$954 \quad 0 \quad \text{Row 5} - \left(\frac{38}{7} \div \frac{-18}{7} \right) \text{Row 4} = \text{Row 5} + \frac{19}{9} \text{Row 4}$$

$$i) \quad \frac{38}{7} + \frac{19}{9} \left(\frac{-18}{7} \right) = 0$$

$$ii) \quad \frac{38}{7} + \frac{19}{9} \left(\frac{-4}{7} \right) = \frac{38}{9}$$

$$\text{iii)} \quad \frac{-2}{7} + \frac{19}{9} \left(\frac{33}{7} \right) = \frac{29}{3}$$

$$\text{iv)} \quad \frac{230}{7} + \frac{19}{9} \left(\frac{-113}{7} \right) = \frac{-11}{9}$$

$$\text{Row 5} \rightarrow \left[0 \ 0 \ 0 \ 0 \ \frac{38}{9} \ \frac{29}{3} : \frac{-11}{9} \right]$$

$$\text{i)} \quad \frac{-26}{7} - \frac{13}{9} \left(\frac{-18}{7} \right) = 0 \quad \text{Row 6} - \frac{13}{9} \text{ Row 4}$$

$$\text{ii)} \quad \frac{2}{7} - \frac{13}{9} \left(\frac{-4}{7} \right) = \frac{10}{9}$$

$$\text{iii)} \quad \frac{-13}{7} - \frac{13}{9} \left(\frac{33}{7} \right) = \frac{-26}{3}$$

$$\text{iv)} \quad \frac{-87}{7} - \frac{13}{9} \left(\frac{-113}{7} \right) = \frac{98}{9}$$

$$\text{new Row 6} \left[0 \ 0 \ 0 \ 0 \ \frac{10}{9} \ -\frac{26}{3} : \frac{98}{9} \right]$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & 1 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{33}{7} & -11 \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{29}{3} & -\frac{11}{9} \\ 0 & 0 & 0 & 0 & \frac{10}{9} & -\frac{26}{3} & \frac{98}{9} \end{bmatrix}$$

$$a_{65} \cdot 0 \quad \text{Row 6} - \left(\frac{10}{9} \div \frac{38}{9} \right) \text{Row 5} \\ = \text{Row 6} - \frac{10}{38} \text{Row 5}$$

$$\text{i)} \quad \frac{10}{9} - \frac{10}{38} \left(\frac{38}{9} \right) = 0$$

$$\text{ii)} \quad \frac{-26}{3} - \frac{10}{38} \left(\frac{29}{3} \right) = \frac{-213}{19}$$

$$\text{iii)} \quad \frac{98}{9} - \frac{10}{38} \left(\frac{-11}{9} \right) = \frac{213}{19}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & 1 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{33}{7} & -11 \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{29}{3} & -\frac{11}{9} \\ 0 & 0 & 0 & 0 & 0 & \frac{-213}{19} & \frac{213}{19} \end{bmatrix}$$

Back substitution

$$i) \quad \frac{-213}{19} T_c = \frac{213}{19}$$

$$T_c = \frac{213}{19} \div \frac{-213}{19} = -1$$

$$ii) \quad \frac{29}{3} (-1) + \frac{38}{9} T_5 = \frac{-11}{9}$$

$$T_5 = \frac{\frac{-11}{9} + \frac{29}{3}}{\frac{38}{9}}$$

$$T_5 = \frac{76}{9} \div \frac{38}{9}$$

$$T_5 = \frac{76}{9} \times \frac{9}{38}$$

$$T_5 = 2$$

$$iii) \quad \frac{-18}{7} T_4 - \frac{4}{7} T_5 + \frac{33}{7} T_c = \frac{-113}{9}$$

$$\frac{-18}{7} T_4 - \frac{4}{7} (2) + \frac{33}{7} (-1) = \frac{-113}{9}$$

$$T_4 = \frac{\frac{-113}{9} + \frac{33}{7} + \frac{8}{7}}{\frac{-18}{7}}$$

$$T_4 = 4$$

$$iv) \quad \frac{7}{3} T_3 - 2T_4 - \frac{13}{3} T_5 + \frac{4}{3} T_c = -11$$

$$\frac{7}{3} (T_3) - 2(4) - \frac{13}{3} (2) + \frac{4}{3} (-1) = -11$$

$$T_3 = \frac{-11 + \frac{4}{3} + \frac{26}{3} + 8}{\frac{7}{3}}$$

$$T_3 = 3$$

$$u) \quad -3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$$

$$-3T_2 + 5(3) + 0(4) - 5(2) - 1(-1) = 12$$

$$T_2 = \frac{12 - 1 + 10 - 15}{-3}$$

$$T_2 = -2$$

$$vi) \quad T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 + (-2) - 2(3) + 4 + 3(2) - (-1) = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 + 3 = 4$$

$$T_1 = 4 - 3$$

$$T_1 = 1 //$$

$$T_6 = -1$$

$$T_5 = 2$$

$$T_4 = 4$$

$$T_3 = 3$$

$$T_2 = -2$$

$$T_1 = 1$$

