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15/EN904/056

ELECT / ELECT

$$\begin{cases}
 T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\
 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\
 T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\
 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\
 -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 5T_6 = 16 \\
 4T_1 + 8T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -21
 \end{cases}$$

The Augmented Matrix

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 2 & -1 & 1 & 2 & 1 & -3 \\
 1 & 3 & -3 & -1 & 2 & 1 \\
 5 & 2 & -1 & -1 & 2 & 1 \\
 -3 & -1 & 2 & 3 & 1 & 3 \\
 4 & 3 & 1 & -6 & -3 & -2
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 20 \\
 -15 \\
 -3 \\
 16 \\
 -21
 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 1 & -2 & 1 & 3 & -1 \\
 2 - (2/1) & -1 - (1/1) & 1 - (-2/1) - 2 & 2 - (2/1) & 1 - (3/1) & -3 - (-1/1) \\
 1 - (1/1) & 3 - (3/1) & -3 - (-2/1) - 2 & -1 - (1/1) & 2 - (3/1) & 1 - (-1/1) \\
 5 - (5/1) & 2 - (2/1) & -1 - (-2/1) - 2 & -1 - (1/1) & 2 - (3/1) & 1 - (-1/1) \\
 -3 - (-3/1) & -1 - (-3/1) & 2 - (-2/1) - 2 & 3 - (3/1) & 1 - (-3/1) & 3 - (-1/1) \\
 4 - (4/1) & 3 - (3/1) & 1 - (-2/1) - 2 & -6 - (6/1) & -3 - (-3/1) & -2 - (-1/1)
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}$$

$$\begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & -2 & 5 & 0 & -5 & -1 \\
 0 & 2 & -1 & -2 & -1 & 2 \\
 0 & -3 & 9 & -6 & -13 & 6 \\
 0 & 2 & -9 & 6 & 10 & 0 \\
 0 & 1 & 9 & -10 & -15 & 2
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 T_3 \\
 T_4 \\
 T_5 \\
 T_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 4 \\
 12 \\
 -19 \\
 -23 \\
 28 \\
 42
 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 - (-\frac{2}{3}) \cdot 3 & -5 - (-\frac{2}{3}) \cdot 5 & -2 - (-\frac{2}{3}) \cdot 0 & -1 - (-\frac{2}{3}) \cdot 5 & 2 - (-\frac{2}{3}) \cdot 1 \\ 0 & -3 - (-\frac{3}{3}) \cdot 3 & 7 - (-\frac{3}{3}) \cdot 5 & -6 - (-\frac{3}{3}) \cdot 0 & -13 - (-\frac{3}{3}) \cdot 5 & 6 - (-\frac{3}{3}) \cdot 1 \\ 0 & 2 - (\frac{2}{3}) \cdot 3 & -4 - (\frac{2}{3}) \cdot 5 & 6 - (\frac{2}{3}) \cdot 0 & 16 - (\frac{2}{3}) \cdot 5 & 0 - (\frac{2}{3}) \cdot 1 \\ 0 & -1 - (-\frac{1}{3}) \cdot 3 & 9 - (-\frac{1}{3}) \cdot 5 & -10 - (-\frac{1}{3}) & -15 - (-\frac{1}{3}) \cdot 5 & 2 - (-\frac{1}{3}) \cdot 1 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix}$$

$$\begin{bmatrix} 4 \\ 12 \\ -19 - (-\frac{2}{3}) \cdot 12 \\ -23 - (-\frac{3}{3}) \cdot 12 \\ 28 - (\frac{2}{3}) \cdot 12 \\ -43 - (-\frac{1}{3}) \cdot 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & -1.4 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -0.7 & 6 & 6.7 & -0.7 \\ 0 & 0 & 7.4 & -6 & -13.4 & 2.4 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} \begin{matrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ -47 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & -1.4 \\ 0 & 0 & 4 \cdot (\frac{4}{2.4}) \cdot 2.4 & -6 - (\frac{4}{2.4}) \cdot 2 & -8 - (\frac{4}{2.4}) \cdot 4.4 & 7 - (\frac{4}{2.4}) \cdot 1.4 \\ 0 & 0 & -0.7 \cdot (\frac{0.7}{2.4}) \cdot 2.4 & 6 - (\frac{0.7}{2.4}) \cdot 2 & 6.7 - (\frac{0.7}{2.4}) \cdot 4.4 & -0.7 \cdot (\frac{0.7}{2.4}) \cdot 1.4 \\ 0 & 0 & 7.4 \cdot (\frac{7.4}{2.4}) \cdot 2.4 & -6 - (\frac{7.4}{2.4}) \cdot 2 & -13.4 - (\frac{7.4}{2.4}) \cdot 4.4 & 2.4 - (\frac{7.4}{2.4}) \cdot 1.4 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \cdot (\frac{4}{2.4}) \cdot (-11) \\ 36 - (\frac{0.7}{2.4}) \cdot (-11) \\ -47 - (\frac{7.4}{2.4}) \cdot (-11) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\ 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\ 0 & 0 & 0 & (5.42) & 5.42 & -0.29 \\ 0 & 0 & 0 & (-3.82) & (0.17) & -1.91 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ 32.79 \\ -13.08 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\ 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\ 0 & 0 & 0 & 5.42 & -\left(\frac{5.42}{-2.7}\right) \cdot 2.7 & -2.7 \cdot 5.42 - \left(\frac{5.42}{-2.7}\right) \cdot 0.7 - 0.29 \cdot \left(\frac{5.42}{-2.7}\right) \\ 0 & 0 & 0 & -3.82 & -\left(\frac{3.82}{-2.7}\right) \cdot 2.7 & -2.7 \cdot 0.17 - \left(\frac{-3.82}{-2.7}\right) \cdot 0.7 - 1.91 \cdot \left(\frac{3.82}{-2.7}\right) \cdot 2.7 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ 32.79 - \left(\frac{5.42}{-2.7}\right) \cdot (-16.7) \\ -13.08 - \left(\frac{-3.82}{-2.7}\right) \cdot (-16.7) \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\ 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\ 0 & 0 & 0 & 0 & 4.01 & 9.2 \\ 0 & 0 & 0 & 0 & (1.16) & -8.59 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ -0.74 \\ 10.71 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\ 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\ 0 & 0 & 0 & 0 & 4.01 & 9.2 \\ 0 & 0 & 0 & 0 & 1.16 & -\left(\frac{1.16}{4.01}\right) \cdot 4.01 - 8.59 - \left(\frac{1.16}{4.01}\right) \cdot 9.2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ -0.74 \\ 10.7 - \left( \frac{11.16}{4.01} \right) (-0.74) \end{bmatrix}$$

The final matrix will be:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.4 & -2 & -4.4 & 1.4 \\ 0 & 0 & 0 & -2.7 & -0.7 & 4.7 \\ 0 & 0 & 0 & 0 & 4.01 & 9.2 \\ 0 & 0 & 0 & 0 & 0 & -11.3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.7 \\ -0.74 \\ 10.92 \end{bmatrix}$$

From the above matrix, we can solve for the temperatures  $T_1, T_2, T_3, T_4, T_5, T_6$  resp.

$$-11.36 T_6 = 10.92$$

$$T_6 = - \frac{10.92}{11.3}$$

$$T_6 = -0.966 \text{ --- (i)}$$

$$4.01 T_5 + 9.2 T_6 = -0.74$$

$$T_5 = \frac{-0.74 - 9.2(-0.966)}{4.01}$$

$$T_5 = \frac{8.1472}{4.01} = 2.031 \text{ --- (ii)}$$

$$-2.7 T_4 - 0.7 T_5 + 4.7 T_6 = -16.7$$

$$-2.7 T_4 - 0.7(2.031) + 4.7(-0.966) = -16.7$$

$$-2.7 T_4 - 1.4217 - 4.5462 = -16.7$$

$$-2.7 T_4 = -16.7 + 1.4217 + 4.5462$$

$$-2.7 T_4 = -10.7381$$

$$T_4 = \frac{10.7381}{2.7} = 3.977 \text{ --- (iii)}$$

$$2.4T_3 - 2T_4 - 4.4T_5 + 1.4T_6 = -11$$

$$2.4T_3 - 2(3.977) - 4.4(2.031) + 1.4(-0.966) = -11$$

$$2.4T_3 - 7.954 - 8.9364 - 1.3524 = -11$$

$$2.4T_3 = -11 + 18.2428$$

$$T_3 = \frac{7.2428}{2.4}$$

$$T_3 = 3.017 \dots \textcircled{4}$$

$$-3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$$

$$-3T_2 + 5(3.017) + 0 - 5(2.031) - 1(-0.966) = 12$$

$$-3T_2 + 15.085 - 10.155 + 0.966 = 12$$

$$-3T_2 = 6.104$$

$$T_2 = \frac{6.104}{3}$$

$$T_2 = -2.034 \dots \textcircled{5}$$

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - 3T_6 = 4$$

$$T_1 + (-2.034) - 2(3.017) + (3.977) + 3(2.031) + 0.966 = 4$$

$$T_1 - 2.034 - 6.034 + 3.977 + 6.093 + 0.966 = 4$$

$$T_1 = 1.032 \dots \textcircled{6}$$

From eqn ① to ⑥ resp.

$$T_1 = 1.032^\circ\text{C}$$

$$T_2 = -2.034^\circ\text{C}$$

$$T_3 = 3.017^\circ\text{C}$$

$$T_4 = 3.977^\circ\text{C}$$

$$T_5 = 2.031^\circ\text{C}$$

$$T_6 = -0.966^\circ\text{C}$$