

UNDIE FIDELIS BEWAPUTE
15/ENG03/033

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 + T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 - 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$$

Solution

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 - \frac{2}{1} \times 1 & -1 - \frac{2}{1} \times 1 & 1 - \frac{2}{1}(-2) & 2 - \frac{2}{1}(1) & 1 - \frac{2}{1}(3) & -3 - \frac{2}{1}(-1) \\ 1 - \frac{1}{1} \times 1 & 3 - \frac{1}{1} \times 1 & -3 - \frac{1}{1}(-2) & -1 - \frac{1}{1}(1) & 2 - \frac{1}{1}(3) & 1 - \frac{1}{1}(-1) \\ 5 - \frac{5}{1} \times 1 & 2 - \frac{5}{1} \times 1 & -1 - \frac{5}{1}(-2) & -1 - \frac{5}{1}(1) & 2 - \frac{5}{1}(3) & 1 - \frac{5}{1}(-1) \\ -3 \times \frac{(-3)}{1} \times 1 & -1 - \frac{(-3)}{1} \times 1 & 2 - \frac{(-3)}{1}(-2) & 3 - \frac{(-3)}{1}(1) & 1 - \frac{(-3)}{1}(3) & 3 - \frac{(-3)}{1}(-1) \\ 4 - \frac{4}{1} \times 1 & 4 - \frac{4}{1} \times 1 & 1 - \frac{4}{1}(-2) & -6 - \frac{4}{1}(1) & -3 - \frac{4}{1}(3) & -2 - \frac{4}{1}(-1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 - \frac{2}{1}(4) \\ -15 - \frac{1}{1}(4) \\ -3 - \frac{5}{1}(4) \\ 16 - \frac{(-3)}{1}(4) \\ -27 - \frac{4}{1}(4) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & -2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & 10 & -15 & 2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2-\frac{2}{3}(3) & -1-\frac{2}{3}(5) & -2-\frac{2}{3}(0) & -1-\frac{2}{3}(-5) & 2-\frac{2}{3}(2) \\ 0 & -3-\frac{(-3)}{2}(3) & 9-\frac{-3}{2}(5) & -6-\frac{-3}{2}(0) & -13-\frac{-3}{2}(-5) & 6-\frac{-3}{2}(-1) \\ 0 & 2-\frac{2}{3}(3) & -4-\frac{2}{3}(5) & 6-\frac{2}{3}(0) & 10-\frac{2}{3}(-5) & 0-\frac{2}{3}(-1) \\ 0 & -1-\frac{-1}{3}(3) & 9-\frac{-1}{3}(5) & -10-\frac{-1}{3}(0) & -15-\frac{-1}{3}(-5) & 2-\frac{-1}{3}(-1) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19-\frac{2}{3}(12) \\ -23-\frac{-3}{2}(12) \\ 28-\frac{2}{3}(12) \\ -43-\frac{1}{3}(12) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -\frac{2}{3} & 6 & \frac{20}{3} & -\frac{2}{3} \\ 0 & 0 & \frac{22}{3} & -10 & -\frac{40}{3} & \frac{7}{3} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ -47 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & 4-\frac{4}{\frac{7}{3}}(\frac{7}{3}) & -6-\frac{4}{\frac{7}{3}}(-2) & -8-\frac{4}{\frac{7}{3}}(-\frac{13}{3}) & 7-\frac{4}{\frac{7}{3}}(\frac{4}{3}) \\ 0 & 0 & \frac{2}{3}-\frac{2}{\frac{7}{3}}(\frac{7}{3}) & 6-\frac{2}{\frac{7}{3}}(-2) & \frac{20}{3}-\frac{2}{\frac{7}{3}}(-\frac{13}{3}) & -\frac{2}{3}-\frac{2}{\frac{7}{3}}(\frac{4}{3}) \\ 0 & 0 & \frac{22}{3}-\frac{22}{\frac{7}{3}}(\frac{7}{3}) & -10-\frac{22}{\frac{7}{3}}(-2) & -\frac{40}{3}-\frac{22}{\frac{7}{3}}(-\frac{13}{3}) & \frac{7}{3}-\frac{22}{\frac{7}{3}}(\frac{4}{3}) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35-\frac{4}{\frac{7}{3}}(-11) \\ 36-\frac{2}{\frac{7}{3}}(-11) \\ -47-\frac{22}{\frac{7}{3}}(-11) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{38}{7} \\ 0 & 0 & 0 & \frac{38}{7} & \frac{38}{7} & -\frac{2}{7} \\ 0 & 0 & 0 & -\frac{26}{7} & \frac{2}{7} & -\frac{13}{7} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ \frac{230}{7} \\ -8\frac{7}{7} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & \frac{38}{7} - \frac{39}{7} \cdot \frac{1}{19} \left(\frac{18}{7} \right) & \frac{38}{7} - \frac{39}{7} \cdot \frac{1}{19} \left(-\frac{4}{7} \right) & -\frac{2}{7} - \frac{39}{7} \cdot \frac{1}{19} \left(\frac{33}{7} \right) \\ 0 & 0 & 0 & -\frac{26}{7} - \left(\frac{34}{7} \right) \left(-\frac{18}{7} \right) & \frac{2}{7} - \frac{24}{7} \cdot \frac{1}{19} \left(-\frac{4}{7} \right) & -\frac{13}{7} - \frac{24}{7} \cdot \frac{1}{19} \left(\frac{33}{7} \right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ \frac{230}{7} - \frac{39}{7} \cdot \frac{1}{19} \left(\frac{113}{7} \right) \\ -\frac{81}{7} - \frac{26}{7} \cdot \frac{1}{19} \left(-\frac{113}{7} \right) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 10/9 & -26/3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ -11/9 \\ 98/9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & \frac{10}{9} - \frac{19}{9} \cdot \frac{1}{38} \left(\frac{38}{9} \right) & -\frac{26}{3} - \frac{19}{9} \cdot \frac{1}{38} \left(\frac{29}{3} \right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ -\frac{11}{9} \\ \frac{98}{9} - \frac{19}{9} \cdot \frac{1}{38} \left(-\frac{11}{9} \right) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 \\ 0 & 0 & 0 & 0 & 0 & -\frac{213}{19} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ -\frac{11}{9} \\ \frac{213}{19} \end{bmatrix}$$

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \quad \text{--- (1)}$$

$$-3T_2 + 5T_3 - 5T_5 - T_6 = 12 \quad \text{--- (2)}$$

$$\frac{7}{3}T_3 - 2T_4 - \frac{13}{3}T_5 + \frac{4}{3}T_6 = -11 \quad \text{--- (3)}$$

$$-\frac{18}{7}T_4 - \frac{4}{7}T_5 + \frac{33}{7}T_6 = -\frac{113}{7} \quad \text{--- (4)}$$

$$\frac{38}{9}T_5 + \frac{29}{3}T_6 = -\frac{11}{9} \quad \text{--- (5)}$$

$$-\frac{213}{19}T_6 = \frac{213}{19} \quad \text{--- (6)}$$

From equation 6

$$\frac{-213}{19}T_6 = \frac{213}{19}$$

$$T_6 = \frac{213}{19} \times \frac{-19}{213}$$

$$T_6 = -1$$

Sub $T_6 = -1$ into eqn 5

$$\frac{38}{9}T_5 + \frac{29}{3}(-1) = -\frac{11}{9}$$

$$\frac{38}{9}T_5 - \frac{29}{3} = -\frac{11}{9}$$

$$\frac{38}{9}T_5 = -\frac{11}{9} + \frac{29}{3}$$

$$\frac{38}{9}T_5 = \frac{76}{9}$$

$$T_5 = 2$$

Sub $T_6 = -1, T_5 = 2$ into eqn 4

$$-\frac{18}{7}T_4 - \frac{4}{7}(2) + \frac{33}{7}(-1) = -\frac{113}{7}$$

$$-\frac{18}{7}T_4 - \frac{8}{7} - \frac{33}{7} = -\frac{113}{7}$$

$$-\frac{18}{7}T_4 = -\frac{113}{7} + \frac{8}{7} + \frac{33}{7}$$

$$\frac{-18}{7}T_4 = -\frac{92}{7}$$

$$T_4 = 4$$

Sub $T_6 = -1, T_5 = 2, T_4 = 4$ in eqn 3

$$\frac{7}{3}T_3 - 2(4) - \frac{13}{3}(2) + \frac{4}{3}(-1) = -11$$

$$\frac{7}{3}T_3 - 8 - \frac{26}{3} - \frac{4}{3} = -11$$

$$\frac{7}{3}T_3 = -11 + 8 + \frac{26}{3} + \frac{4}{3}$$

$$\frac{7}{3}T_3 = 7$$

$$T_3 = 3$$

Sub $T_6 = -1, T_5 = 2, T_4 = 4, T_3 = 3$ in eqn 2

$$-3T_2 + 5(3) + 0 - 5(2) - (-1) = 12$$

$$-3T_2 + 15 - 10 + 1 = 12$$

$$-3T_2 = 12 - 15 + 10 - 1$$

$$-3T_2 = 6$$

$$T_2 = \frac{6}{-3}$$

$$T_2 = -2$$

Sub $T_6 = -1$, $T_5 = 2$, $T_4 = 4$, $T_3 = 3$, $T_2 = 2$ into eqn 1

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 + (-2) - 2(3) + 4 + 3(2) - (-1) = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 = 4 + 2 + 6 - 4 - 6 - 1$$

$$T_1 = 1$$

$$T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$