

Manually Use of Gauss Elimination Method

$$\begin{aligned} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 &= 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 &= 20 \\ T_1 + 3T_2 - 8T_3 - T_4 + 4T_5 + T_6 &= -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 &= -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 &= 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 5T_5 - 2T_6 &= -27 \end{aligned}$$

Solution

1	1	-2	1	3	-1	T_1	4
2	-1	1	2	1	-3	T_2	20
3	1	3	-3	-1	2	T_3	-15
4	5	2	-1	-1	2	T_4	-3
5	-3	-1	2	3	1	T_5	16
6	4	3	1	-6	-2	T_6	-27

Augmented Matrix

1	1	-2	1	3	-1	4
2	-1	1	2	1	-3	20
3	1	3	-3	-1	2	-15
4	5	2	-1	-1	2	-3
5	-3	-1	2	3	1	16
6	4	3	1	-6	-2	-27

To make a_{11} 0, Row 2 $- \frac{1}{1} \times$ Row 1

$$\Rightarrow 2 - (-1 \times 1) = 0; (i) -1 - \frac{1}{1}(-2) = -3 \Rightarrow 1 - \frac{1}{1}(-3) = 5$$

$$(ii) 2 - \frac{1}{1}(1) = 0; (iii) 1 - \frac{1}{1}(3) = -2 \Rightarrow (iv) -1 - \frac{1}{1}(2) = -3$$

$$(v) 20 - \frac{1}{1}(4) = 16$$

$$\text{Row 2 becomes } [0 \ -3 \ 5 \ 0 \ -5 \ -1 : 16]$$

To make a_{22} 0, Row 3 $- \frac{1}{-3} \times$ Row 2

$$\Rightarrow 1 - 1(1) = 0; (i) 3 - 1(1) = 2; (ii) -3 - 1(-2) = -1; (iii) -1 - 1(2) = -3$$

$$(iv) 2 - 1(1) = 1; (v) 1 - 1(-1) = 2; (vi) -15 - 1(16) = -31$$

$$(vii) 4 - 1(-1) = 2; (viii) -15 - 1(16) = -31$$

$$\text{Row 3 becomes } [0 \ 2 \ -1 \ -2 \ -1 \ 2 : -31]$$

To make a_{31} 0, Row 4 $- \frac{5}{1} \times$ Row 3

$$(i) 5 - 5(1) = 0; (ii) 2 - 5(2) = -8; (iii) -1 - 5(-1) = 4; (iv) -1 - 5(2) = -11$$

$$(v) 2 - 5(3) = -13; (vi) 1 - 5(-1) = 6; (vii) -3 - 5(2) = -13$$

$$\text{Row 4 becomes } [0 \ -3 \ 9 \ -6 \ -13 \ 6 : -23]$$

To make a_{21} 0; we have $\text{Row } 2 + 2 \times \text{Row } 1$
 $2 + 2(3) = 0$; (ii) $-1 + 2(-2) = -5$; (iii) $0 + 2(5) = 10$
 $1 + 2(4) = 9$; (iv) $3 + 2(0) = 6$; (v) $16 + 2(0) = 32$
 The new Row 2 = $[0 \ 2 \ -5 \ 6 \ 10 \ 32]$

To make a_{31} 0; we have $\text{Row } 3 - 4 \times \text{Row } 1$
 $4 - 4(3) = 0$; (ii) $3 - 4(-2) = 11$; (iii) $9 - 4(5) = -11$
 $16 - 4(0) = 16$; (v) $2 - 4(0) = 2$; (vi) $-23 - 4(0) = -23$
 New Row 3 = $[0 \ -1 \ 11 \ -11 \ 2 \ -23]$

The new set of matrix:

1	1	-2	1	3	-1	4
0	2	-5	6	10	32	16
0	-1	11	-11	2	-23	16
0	-3	9	-6	-13	6	-23
0	2	-1	6	10	0	25
0	1	9	10	-15	2	-43

To make a_{32} 0; we have $\text{Row } 3 - \frac{1}{2} \times \text{Row } 2$
 $2 - \frac{1}{2}(2) = 0$; (ii) $-1 - \frac{1}{2}(-5) = \frac{3}{2}$; (iii) $11 - \frac{1}{2}(11) = \frac{11}{2}$
 $-11 - \frac{1}{2}(-11) = -\frac{11}{2}$; (v) $2 - \frac{1}{2}(2) = 1$
 $-23 - \frac{1}{2}(-23) = -\frac{23}{2}$

The new Row 3 = $[0 \ 0 \ \frac{7}{2} \ -2 \ -\frac{11}{2} \ \frac{4}{2} \ -11]$

To make a_{41} 0; we have $\text{Row } 4 - (-\frac{2}{3}) \times \text{Row } 2 = \text{Row } 4 + \frac{2}{3} \times \text{Row } 2$
 $-3 + \frac{2}{3}(2) = 0$; (ii) $9 + \frac{2}{3}(-5) = 4$; (iii) $-6 + \frac{2}{3}(11) = -\frac{2}{3}$
 $6 + \frac{2}{3}(-6) = 2$; (v) $23 + \frac{2}{3}(10) = -\frac{35}{3}$

New Row 4 = $[0 \ 0 \ 4 \ -6 \ -8 \ \frac{7}{3} \ -35]$

To make a_{51} 0; we have $\text{Row } 5 - \frac{2}{3} \times \text{Row } 2$
 $2 - \frac{2}{3}(2) = 0$; (ii) $-4 + \frac{2}{3}(-5) = -\frac{14}{3}$; (iii) $6 + \frac{2}{3}(11) = 6$
 $10 + \frac{2}{3}(10) = \frac{40}{3}$; (v) $0 + \frac{2}{3}(-1) = -\frac{2}{3}$; (vi) $28 + \frac{2}{3}(32) = 36$

The New Row 5 = $[0 \ 0 \ -\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} \ 36]$

To make a_{61} 0; we have; $\text{Row } 6 - \frac{1}{3} \times \text{Row } 2$

$-1 - \frac{1}{3}(2) = 0$; (ii) $9 - \frac{1}{3}(5) = \frac{22}{3}$; (iii) $-10 - \frac{1}{3}(11) = -\frac{31}{3}$
 $-15 - \frac{1}{3}(-5) = -\frac{40}{3}$; (v) $2 - \frac{1}{3}(-1) = \frac{7}{3}$; (vi) $-43 - \frac{1}{3}(2) = -43$

The New Row 6 = $[0 \ 0 \ \frac{22}{3} \ -10 \ -\frac{40}{3} \ \frac{7}{3} \ -43]$

The above set of Augmented matrix is:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & : & -11 \\ 0 & 0 & 4 & -6 & -3 & 7 & : & -35 \\ 0 & 0 & -2/3 & 6 & -2/3 & -7/3 & : & 34 \\ 0 & 0 & 2/3 & -10 & -40/3 & 2/3 & : & -92 \end{bmatrix}$$

To make a_{22} 0, we have Row 4 $-(4/3) \times$ Row 3

$$\text{Row 4} = -\frac{4}{3} \times \text{Row 3}$$

$$\text{Row 4} = -\frac{4}{3} \left(\frac{7}{3} \right) = -\frac{28}{9}; -6 - \left(-\frac{12}{3} \right) (-2) = -\frac{11}{3}; -35 - \left(-\frac{12}{3} \right) (-11) = -\frac{113}{3}$$

$$(iv) 7 - \left(\frac{12}{3} \right) \left(\frac{4}{3} \right) = \frac{23}{3}; (v) -35 - \left(\frac{12}{3} \right) (-11) = -\frac{113}{3}$$

$$\text{New Row 4} = \begin{bmatrix} 0 & 0 & 0 & -11/3 & -4/3 & 23/3 & : & -113/3 \end{bmatrix}$$

To make a_{33} 0, we have Row 5 $-(9/7/3) \times$ Row 3

$$= \text{Row 5} \times \frac{2}{3} \text{ Row 3}$$

$$\text{Row 5} = \frac{2}{3} \left(\frac{7}{3} \right) = \frac{14}{9}; (i) 6 + \frac{2}{3} (-2) = \frac{10}{3}; (ii) \frac{11}{3} + \frac{2}{3} \left(-\frac{13}{3} \right) = \frac{23}{9}$$

$$(iii) -\frac{11}{3} + \frac{2}{3} \left(\frac{4}{3} \right) = -\frac{22}{9}; (iv) 34 + \frac{2}{3} (-11) = \frac{230}{3}$$

$$\text{New Row 5} = \begin{bmatrix} 0 & 0 & 0 & 39/2 & 38/3 & -2/3 & : & 230/3 \end{bmatrix}$$

To make a_{63} 0, we have Row 6 $-(2/3/7/3) \times$ Row 3 $=$ Row 6

$$\text{Row 6} = -\frac{2}{3} \left(\frac{7}{3} \right) = -\frac{14}{9}; (i) -10 - \frac{2}{3} (-2) = -\frac{26}{3}; (ii) -\frac{40}{3} - \frac{2}{3} \left(-\frac{13}{3} \right) = -\frac{82}{9}$$

$$(iii) \frac{2}{3} - \frac{2}{3} \left(\frac{4}{3} \right) = -\frac{2}{9}; (iv) -92 - \frac{2}{3} (-11) = -\frac{82}{3}$$

$$\text{The new Row 6} = \begin{bmatrix} 0 & 0 & 0 & -26/3 & 2/3 & -82/3 & : & -82/3 \end{bmatrix}$$

∴ This is the above set of matrices is:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & : & -11 \\ 0 & 0 & 0 & -11/3 & -4/3 & 23/3 & : & -113/3 \\ 0 & 0 & 0 & 39/2 & 38/3 & -2/3 & : & 230/3 \\ 0 & 0 & 0 & -26/3 & 2/3 & -82/3 & : & -82/3 \end{bmatrix}$$

To make a_{66} 0, we have Row 5 $-(39/2/11/3) \times$ Row 4

$$= \text{Row 5} + \frac{19}{3} \text{ Row 4}$$

$$\text{Row 5} = \frac{39}{2} + \frac{19}{3} \left(-\frac{11}{3} \right) = 0; (i) \frac{38}{3} + \frac{19}{3} \left(-\frac{4}{3} \right) = \frac{38}{9}; (ii) -\frac{2}{3} + \frac{19}{3} \left(\frac{23}{3} \right) = \frac{436}{9}$$

$$(iii) -\frac{82}{3} + \frac{19}{3} \left(-\frac{113}{3} \right) = -\frac{2113}{9}; (iv) \frac{230}{3} + \frac{19}{3} \left(-\frac{113}{3} \right) = -\frac{11}{3}$$

To make Row 5 = $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -2 & -1 \end{bmatrix}$
 To make Row 6 = $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -2 & -1 \end{bmatrix}$
 To make Row 6 = $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -2 & -1 \end{bmatrix}$

The augmented matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2/3 & -2 & -1/3 & 2/3 & -11 \\ 0 & 0 & 0 & -4/3 & -4/3 & 2/3 & -11/3 \\ 0 & 0 & 0 & 0 & 3/4 & 2/3 & -11/4 \\ 0 & 0 & 0 & 0 & 10/3 & -2/3 & 2/3 \end{bmatrix}$$

To make $a_{55} = 0$: Row 6 = $(\frac{10}{3} \div \frac{3}{4}) \times \text{Row 5}$
 $\Rightarrow \text{Row 6} = \frac{40}{9} \times \text{Row 5}$
 $\frac{10}{3} - \frac{40}{9}(\frac{3}{4}) = 0$; (ii) $-2/3 - \frac{40}{9}(\frac{2}{3}) = -\frac{213}{9}$
 $\frac{2}{3} - \frac{40}{9}(\frac{2}{3}) = -\frac{213}{9}$

Augmented matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2/3 & -2 & -1/3 & 2/3 & -11 \\ 0 & 0 & 0 & -4/3 & -4/3 & 2/3 & -11/3 \\ 0 & 0 & 0 & 0 & 3/4 & 2/3 & -11/4 \\ 0 & 0 & 0 & 0 & 0 & -213/9 & 213/9 \end{bmatrix}$$

Back substitution

(i) $-\frac{213}{9} T_6 = \frac{213}{9}$
 $T_6 = \frac{213}{9} / -\frac{213}{9} = -1$
 (ii) $\frac{29}{3}(-1) + \frac{35}{4} T_5 = -\frac{110}{4}$
 $T_5 = \frac{-110/4 + 29/3}{30/4}$

$T_5 = 2$

(iii) $-\frac{15}{4} T_4 - \frac{4}{3} T_5 + \frac{33}{4} T_6 = -\frac{113}{4}$
 $\Rightarrow -\frac{15}{4} T_4 - \frac{4}{3}(2) + \frac{33}{4}(-1) = -\frac{113}{4}$
 $T_4 = \frac{-113/4 + 8/3 + 33/4}{-15/4}$

$T_4 = 4$

$$\begin{aligned}
 (iv) \quad & 4T_2 - 2T_3 - 11T_4 + 7T_5 = -11 \\
 & 2T_2 = 2(4) - 11(2) + 7(3) + 1(1) = -11 \\
 & \therefore T_2 = \frac{-11 + \frac{7}{3} + \frac{26}{3} + 1}{\frac{2}{3}} \\
 & \qquad \qquad \qquad \frac{7}{3}
 \end{aligned}$$

$$T_2 = 3\frac{1}{3}$$

$$\begin{aligned}
 (v) \quad & -3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12 \\
 & -3(4) + 5(6) + 0(4) - 5(2) - 1(1) = 12 \\
 & T_2 = \frac{12 - 1 + 10 - 15}{-3}
 \end{aligned}$$

$$T_2 = -2$$

$$\begin{aligned}
 (vi) \quad & T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 9 \\
 & T_1 + (-1) - 2(2) + 4 + 3(6) - (-4) = 9 \\
 & T_1 - 2 - 4 + 4 + 18 + 4 = 9 \\
 & T_1 + 3 = 9 \\
 & T_1 = 9 - 3 \\
 & T_1 = 6 //
 \end{aligned}$$

$$\therefore T_1 = 6$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$