

Manually Use of Gauss Elimination Method

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 + 2T_6 = -27$$

Solve

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{bmatrix}$$

To make a_{21} 0, Row 2 $- 2 \times$ Row 1

$$(i) = 2 - (2 \times 1) = 0; (ii) = -1 - 2(1) = -3; (iii) = 1 - 2(-2) = 5$$

$$(iv) = 2 - 2(1) = 0; (v) = 1 - 2(3) = -5; (vi) = -3 - 2(-1) = -1$$

$$(vii) = 20 - 2(4) = 12$$

$$\text{Row 2 becomes; } [0 \quad -3 \quad 5 \quad 0 \quad -5 \quad -1 \quad 12]$$

To make a_{31} 0, Row 3 $- 1 \times$ Row 1

$$(i) = 1 - 1(1) = 0; (ii) = 3 - 1(1) = 2; (iii) = -3 - 1(-2) = -1; (iv) = -1 - 1(1) = -2$$

$$(v) = 2 - 1(3) = -1; (vi) = 1 - 1(-1) = 2; (vii) = -15 - 1(4) = -19$$

$$\text{Row 3 becomes } [0 \quad 2 \quad -1 \quad -2 \quad -1 \quad 2 \quad -19]$$

To make a_{41} 0, Row 4 $- 5 \times$ Row 1

$$(i) = 5 - 5(1) = 0; (ii) = 2 - 5(1) = -3; (iii) = -1 - 5(-2) = 9; (iv) = -1 - 5(1) = -6$$

$$(v) = 2 - 5(3) = -13; (vi) = 1 - 5(-1) = 6; (vii) = -3 - 5(4) = -23$$

$$\text{Row 4 becomes } = [0 \quad -3 \quad 9 \quad -6 \quad -13 \quad 6 \quad -23]$$

16
27/3

To make a_{51} 0; we have Row 5 + 3 × Row 1
 (i) $-3 + 3(1) = 0$ (ii) $-1 + 3(1) = 2$ (iii) $2 + 3(-2) = -4$ (iv) $3 + 3(1) = 6$
 (v) $1 + 3(3) = 10$ (vi) $3 + 3(-1) = 0$ (vii) $16 + 3(4) = 28$
 The new Row 5 = $[0 \ 2 \ -4 \ 6 \ 10 \ 0 : 28]$

To make a_{61} 0; we have Row 6 - $\frac{4}{3}$ × Row 1
 (i) $4 - \frac{4}{3}(1) = \frac{8}{3}$ (ii) $3 - \frac{4}{3}(1) = \frac{5}{3}$ (iii) $1 - \frac{4}{3}(-2) = 9$ (iv) $-6 - \frac{4}{3}(1) = -10$
 (v) $-3 - \frac{4}{3}(3) = -15$ (vi) $-2 - \frac{4}{3}(-1) = 2$ (vii) $-27 - \frac{4}{3}(4) = -43$
 New Row 6 = $[0 \ -1 \ 9 \ -10 \ -15 \ 2 : 43]$

Thus the new set of Matrix;

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 : 4 \\ 0 & -3 & 5 & 0 & -5 & -1 : 12 \\ 0 & 2 & -1 & -2 & -1 & 2 : -19 \\ 0 & -3 & 9 & -6 & -13 & 6 : -23 \\ 0 & 2 & -4 & 6 & 10 & 0 : 28 \\ 0 & -1 & 9 & -10 & -15 & 2 : 43 \end{bmatrix}$$

$$\begin{array}{r} 10 - 10 \cdot 7 \\ -4 + 10 \cdot 30 - 10 = 20 \\ 12 + 10 \cdot 3 = 42 \\ 3 \cdot 2 \cdot 3 = 18 \\ 12 \\ \times 4 \quad 35 \quad 10 \quad 28 \cdot 18 \\ 11 + 8 \quad 12 \cdot 10 = 120 \\ 2 - 2 \\ 6 - 3 = 3 \end{array}$$

To make a_{32} 0; we have Row 3 - $\frac{2}{3}$ × Row 2
 (i) $2 - \frac{2}{3}(-3) = 0$ (ii) $-1 - \frac{2}{3}(5) = \frac{7}{3}$ (iii) $-2 - \frac{2}{3}(0) = -2$ (iv) $-1 - \frac{2}{3}(-5) = \frac{7}{3}$
 (v) $2 - \frac{2}{3}(-3) = 4$ (vi) $-19 - \frac{2}{3}(12) = -11$

The new Row 3 = $[0 \ 0 \ \frac{7}{3} \ -2 \ -\frac{13}{3} \ \frac{4}{3} : -11]$

To make a_{42} 0; we have Row 4 - $\frac{2}{3}$ × Row 2 = Row 4 - $\frac{1}{3}$ × Row 3
 (i) $-3 - \frac{1}{3}(-3) = 0$ (ii) $9 - \frac{1}{3}(5) = 4$ (iii) $-6 - \frac{1}{3}(0) = -6$ (iv) $-13 - \frac{1}{3}(-5) = -8$
 (v) $6 - \frac{1}{3}(-1) = \frac{19}{3}$ (vi) $-23 - \frac{1}{3}(12) = -35$
 New Row 4 = $[0 \ 0 \ 4 \ -6 \ -8 \ \frac{19}{3} : -35]$

To make a_{52} 0; we have Row 5 - $\frac{2}{3}$ × Row 2 = Row 5 + $\frac{2}{3}$ × Row 3
 (i) $2 + \frac{2}{3}(-3) = 0$ (ii) $-4 + \frac{2}{3}(5) = -\frac{2}{3}$ (iii) $6 + \frac{2}{3}(0) = 6$
 (iv) $10 + \frac{2}{3}(-5) = \frac{20}{3}$ (v) $0 + \frac{2}{3}(-1) = -\frac{2}{3}$ (vi) $28 + \frac{2}{3}(12) = 36$
 The new Row 5 = $[0 \ 0 \ -\frac{2}{3} \ 6 \ \frac{20}{3} \ -\frac{2}{3} : 36]$

To make a_{62} 0; we have Row 6 - $\frac{1}{3}$ × Row 2
 (i) $-1 - \frac{1}{3}(-3) = 0$ (ii) $9 - \frac{1}{3}(5) = \frac{22}{3}$ (iii) $-10 - \frac{1}{3}(0) = -10$
 (iv) $-15 - \frac{1}{3}(-5) = -\frac{40}{3}$ (v) $2 - \frac{1}{3}(-1) = \frac{7}{3}$ (vi) $-43 - \frac{1}{3}(12) = -47$
 The new Row 6 = $[0 \ 0 \ \frac{22}{3} \ -10 \ -\frac{40}{3} \ \frac{7}{3} : -47]$

Thus the new

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 5 \\ 0 & 0 & \frac{7}{3} \\ 0 & 0 & 4 \\ 0 & 0 & -\frac{2}{3} \\ 0 & 0 & \frac{22}{3} \end{bmatrix}$$

To make a_{13}

(i) $4 - \frac{12}{7}(\frac{7}{3}) = 0$
 (iv) $7 - \frac{12}{7}(\frac{4}{3}) = \frac{17}{7}$

New Row 4

To make

(i) $-2/3 + 2/7$
 (iv) $-2/3 + 2/7$

New Row

To make

(i) $\frac{22}{3} - 2$
 (iv) $\frac{7}{3} -$

The new

Thus the

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

To make

(i) $\frac{38}{7} + \frac{19}{7}$
 (iv) $\frac{230}{7}$

The new

(i) $-\frac{26}{7}$
 (iv) $-\frac{1}{7}$

The

$$3+3(1)=6$$

$$-4(1) = -10$$

$$= -43$$

$$-\frac{10}{3}$$

$$30 - \frac{10}{3} = \frac{20}{3}$$

$$= \frac{20}{3}$$

$$= \frac{20}{3}$$

$$-\frac{10}{3}$$

$$-\frac{10}{3}$$

$$= \frac{20}{3}$$

$$= \frac{20}{3}$$

$$(i) -1 - \frac{2}{3}(-5) = \frac{13}{3}$$

11]

$$2 = \text{Row 4} - 1(\text{Row 3})$$

$$(iv) -13 - 1(-5) = -8$$

$$\frac{2}{3} \times \text{Row 2}$$

$$= 6$$

$$18 + \frac{2}{3}(12) = 36$$

$$[6]$$

$$-10$$

$$= -47$$

$$47]$$

Thus the New set of Augmented Matrix is:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & -11 \\ 0 & 0 & 4 & -6 & -8 & 7 & -35 \\ 0 & 0 & -\frac{7}{3} & 6 & \frac{20}{3} & -\frac{2}{3} & 36 \\ 0 & 0 & \frac{2}{3} & -10 & -\frac{40}{3} & \frac{7}{3} & -47 \end{bmatrix}$$

$$\frac{2}{3} \times \frac{3}{1}$$

To make a_{43} 0; We have, $\text{Row 4} - \frac{4}{7}(\text{Row 3})$; $\text{Row 4} - \frac{12}{7} \times \text{Row 3}$

$$(i) 4 - \frac{12}{7}(\frac{7}{3}) = 0; (ii) -6 - \frac{12}{7}(-2) = -\frac{18}{7}; (iii) -8 - \frac{12}{7}(-\frac{13}{3}) = -\frac{4}{7};$$

$$(iv) 7 - \frac{12}{7}(\frac{4}{3}) = \frac{38}{7}; (v) -35 - \frac{12}{7}(-11) = -\frac{113}{7}$$

$$\text{New Row 4} = [0 \ 0 \ 0 \ -\frac{18}{7} \ -\frac{4}{7} \ \frac{38}{7} \ -\frac{113}{7}]$$

To make a_{53} 0; We have $\text{Row 5} - \frac{2}{3}(\text{Row 3})$; $\text{Row 5} + \frac{2}{7} \text{Row 3}$

$$(i) -\frac{7}{3} + \frac{2}{7}(\frac{7}{3}) = 0; (ii) 6 + \frac{2}{7}(-2) = \frac{38}{7}; (iii) \frac{20}{3} + \frac{2}{7}(-\frac{13}{3}) = \frac{38}{7}$$

$$(iv) -\frac{2}{3} + \frac{2}{7}(\frac{4}{3}) = -\frac{2}{7}; (v) 36 + \frac{2}{7}(-11) = \frac{230}{7}$$

$$\text{New Row 5} = [0 \ 0 \ 0 \ \frac{38}{7} \ -\frac{2}{7} \ \frac{38}{7} \ \frac{230}{7}]$$

To make a_{63} 0; we have $\text{Row 6} - \frac{2}{3}(\text{Row 3})$; $\text{Row 6} - \frac{20}{7} \text{Row 3}$

$$(i) \frac{2}{3} - \frac{20}{7}(\frac{7}{3}) = 0; (ii) -10 - \frac{20}{7}(-2) = -\frac{26}{7}; (iii) -\frac{40}{3} - \frac{20}{7}(-\frac{13}{3}) = \frac{2}{7};$$

$$(iv) \frac{7}{3} - \frac{20}{7}(\frac{4}{3}) = -\frac{13}{7}; (v) -47 - \frac{20}{7}(-11) = -\frac{87}{7}$$

$$\text{The new Row 6} = [0 \ 0 \ 0 \ -\frac{26}{7} \ \frac{2}{7} \ -\frac{13}{7} \ -\frac{87}{7}]$$

Thus the New set of Matrix is:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & -11 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{38}{7} & -\frac{113}{7} \\ 0 & 0 & 0 & \frac{38}{7} & -\frac{2}{7} & \frac{38}{7} & \frac{230}{7} \\ 0 & 0 & 0 & -\frac{26}{7} & \frac{2}{7} & -\frac{13}{7} & -\frac{87}{7} \end{bmatrix}$$

$$-\frac{26}{7} / -\frac{18}{7}$$

To make a_{54} 0; We have, $\text{Row 5} - \frac{38}{7}(\frac{18}{7}) \times \text{Row 4}$; $\text{Row 5} + \frac{19}{7} \text{Row 4}$

$$(i) \frac{38}{7} + \frac{19}{7}(-\frac{18}{7}) = 0; (ii) \frac{38}{7} + \frac{19}{7}(-\frac{4}{7}) = \frac{38}{7}; (iii) \frac{2}{7} + \frac{19}{7}(\frac{38}{7}) = \frac{29}{7}$$

$$(iv) \frac{230}{7} + \frac{19}{7}(-\frac{113}{7}) = -\frac{11}{7}$$

$$\text{The New Row 5} = [0 \ 0 \ 0 \ 0 \ \frac{38}{7} \ \frac{29}{7} \ -\frac{11}{7}]$$

The New R. To make a_{64} 0; We have, $\text{Row 6} - \frac{13}{9} \text{Row 4}$

$$(i) -\frac{26}{7} - \frac{13}{9}(-\frac{18}{7}) = 0; (ii) \frac{2}{7} - \frac{13}{9}(-\frac{4}{7}) = \frac{12}{7}; (iii) -\frac{13}{7} - \frac{13}{9}(\frac{38}{7}) = -\frac{26}{3}$$

$$(iv) -\frac{87}{7} - \frac{13}{9}(-\frac{113}{7}) = \frac{98}{9}$$

$$\text{The New Row 6} = [0 \ 0 \ 0 \ 0 \ \frac{12}{7} \ -\frac{26}{3} \ \frac{98}{9}]$$

$$\frac{10}{9} \times \frac{1}{38} = \frac{10}{342}$$

The New Augmented Matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 10/9 & -26/3 & 98/9 \end{bmatrix}$$

To make a_{65} 0; Row 6 - $(10/9 \div \frac{38}{9}) \times \text{Row 5}$;

$$\Rightarrow \text{Row 6} - \frac{10}{38} \times \text{Row 5}$$

$$(i) \frac{10}{9} - \frac{10}{38} \left(\frac{38}{9} \right) = 0; (ii) -\frac{26}{3} - \frac{10}{38} \left(\frac{29}{3} \right) = -\frac{213}{19}$$

$$(iii) \frac{98}{9} - \frac{10}{38} \left(-\frac{11}{9} \right) = \frac{213}{19}$$

Back Substitution

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 7/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 0 & -18/7 & -4/7 & 33/7 & -113/7 \\ 0 & 0 & 0 & 0 & 38/9 & 29/3 & -11/9 \\ 0 & 0 & 0 & 0 & 0 & -\frac{213}{19} & \frac{213}{19} \end{bmatrix}$$

Back Substitution

$$(i) -\frac{213}{19} T_6 = \frac{213}{19}$$

$$T_6 = \frac{213}{19} / -\frac{213}{19} = -1$$

$$(ii) \frac{29}{3} (-1) + \frac{38}{9} T_5 = -11/9$$

$$T_5 = \frac{-11/9 + 29/3}{38/9}$$

$$T_5 = 2$$

$$(iii) -18/7 T_4 - 4/7 T_5 + 33/7 T_6 = -113/7$$

$$\Rightarrow -18/7 T_4 - \frac{4}{7} (2) + \frac{33}{7} (-1) = -113/7$$

$$T_4 = \frac{-113/7 + 8/7 + 33/7}{-18/7}$$

$$T_4 = 4$$

$$(iv) \frac{7}{3}T_3 - 2T_4 - \frac{13}{3}T_5 + \frac{4}{3}T_6 = -11$$

$$\frac{7}{3}T_3 - 2(4) - \frac{13}{3}(2) + \frac{4}{3}(-1) = -11$$

$$T_3 = \frac{-11 + \frac{4}{3} + \frac{26}{3} + 8}{\frac{7}{3}}$$

$$T_3 = \underline{\underline{3}}$$

$$(v) -3T_2 + 5T_3 + 0T_4 - 5T_5 - 1T_6 = 12$$

$$-3T_2 + 5(3) + 0(4) - 5(2) - 1(-1) = 12$$

$$T_2 = \frac{12 - 1 + 10 - 15}{-3}$$

$$T_2 = -2$$

$$(vi) T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 + (-2) - 2(3) + 4 + 3(2) - (-1) = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$

$$T_1 + 3 = 4$$

$$T_1 = 4 - 3$$

$$T_1 = \underline{\underline{1}}$$

$$\therefore T_1 = 1$$

$$T_2 = -2$$

$$T_3 = 3$$

$$T_4 = 4$$

$$T_5 = 2$$

$$T_6 = -1$$