

Name: EGIS BLESSING OLUKPA

matric no: 15/ENG071013

Department: Petroleum Engineering

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Assignment 3

Question:

$$T_1 + T_2 - 2T_3 + T_4 + 8T_5 - T_6 = 4$$

$$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$$

$$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$$

$$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$$

$$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$$

$$4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$$

Solution:

Writing the given equation in its matrix form:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 8 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

The Augmented Form:

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 8 & -1 & 4 \\ 2 & -1 & 1 & 2 & 1 & -3 & 20 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 1 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{bmatrix}$$

~~Step 1~~ Step 1  $[Row 2 - \frac{2}{1} Row 1]$

$$2 - 2 \times 1 = 0$$

$$-1 - 2 \times 1 = -3$$

$$1 - 2 \times -2 = 5$$

$$2 - 2 \times 1 = 0$$

$$1 - 2 \times 8 = -15$$

$$-3 - 2 \times -1 = -1$$

$$20 - 2 \times 4 = 12$$

Then modify Row 2.

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 1 & 3 & -3 & -1 & 2 & 1 & -15 \\ 5 & 2 & -1 & -1 & 2 & 7 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{bmatrix}$$

$-19 + \frac{2}{3}(12) = -11$  \*  
Then modify Row 3.

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & & & & & & \end{bmatrix}$$

→ Step 2  $[Row 3 - \frac{1}{3} Row 1]$

$$\begin{aligned} 1 - 1 \times 1 &= 0 \quad \text{---} * \\ 3 - 1 \times 1 &= 2 \quad \text{---} * \\ -3 - 1 \times -2 &= -1 \quad \text{---} * \\ -1 - 1 \times 1 &= -2 \quad \text{---} * \\ 2 - 1 \times 3 &= -1 \quad \text{---} * \\ 1 - 1 \times -1 &= 2 \quad \text{---} * \\ -15 - 1 \times 4 &= -19 \quad \text{---} * \end{aligned}$$

Then modify Row 3.

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 5 & 2 & -1 & -1 & 2 & 7 & -3 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{bmatrix}$$

→ Step 3  $[Row 3 - (\frac{2}{3}) Row 2]$

Row 3 +  $\frac{2}{3} Row 2$ .

$$\begin{aligned} 2 + \frac{2}{3}(-3) &= 0 \quad \text{---} * \\ -1 + \frac{2}{3}(5) &= \frac{7}{3} \quad \text{---} * \\ -2 + \frac{2}{3}(0) &= -2 \quad \text{---} * \\ -1 + \frac{2}{3}(-5) &= -\frac{13}{3} \quad \text{---} * \\ 2 + \frac{2}{3}(-1) &= \frac{4}{3} \end{aligned}$$

→ Step 3  $(Row 4 - \frac{5}{1} Row 1)$

$$\begin{aligned} 5 - \frac{5}{1}(1) &= 0 \quad \text{---} * \\ 2 - 5(1) &= -3 \quad \text{---} * \\ -1 - 5(-2) &= 9 \quad \text{---} * \\ -1 - 5(1) &= -6 \quad \text{---} * \\ 2 - 5(3) &= -13 \quad \text{---} * \\ 1 - 5(-1) &= 6 \quad \text{---} * \\ -3 - 5(4) &= -23 \quad \text{---} * \end{aligned}$$

Then modify Row 4.

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ -3 & -1 & 2 & 3 & 1 & 3 & 16 \\ 4 & 3 & 1 & -6 & -3 & -2 & -27 \end{bmatrix}$$

→ Step 4  $[Row 5 - (-\frac{3}{1}) Row 1]$

$$\begin{aligned} -3 + 3 \times 1 &= 0 \quad \text{---} * \\ -1 + 3(1) &= 2 \quad \text{---} * \\ 2 + 3(-2) &= -4 \quad \text{---} * \\ 3 + 3(1) &= 6 \quad \text{---} * \\ 1 + 3(3) &= 10 \quad \text{---} * \\ 3 + 3(-1) &= 0 \quad \text{---} * \\ 16 + 3(4) &= 28 \quad \text{---} * \end{aligned}$$

then modify row 5

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 4 & 3 & 1 & -6 & -3 & -2 & -23 \end{bmatrix}$$

then modify row 3

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{1}{3} & -2 & -1\frac{1}{3} & \frac{4}{3} & -1 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & 10 & -15 & 2 & -4 \end{bmatrix}$$

→ step 5 [row 6 - 4 row 1]

$$\begin{aligned} 4 - 4(1) &= 0 \quad - \quad * \\ 3 - 4(1) &= -1 \quad - \quad * \\ 1 - 4(-2) &= 9 \quad - \quad * \\ -6 - 4(1) &= -10 \quad - \quad * \\ -3 - 4(3) &= -15 \quad - \quad * \\ -2 - 4(-1) &= 2 \quad - \quad * \\ -23 - 4(4) &= -43 \quad - \quad * \end{aligned}$$

→ step 7 [row 4 - (3/2) row 3]

$$\begin{aligned} -3 + \frac{3}{2}(\frac{1}{3}) &= 0 \quad - \quad * \\ 9 + \frac{3}{2}(-2) &= 15/2 \quad - \quad * \\ -6 + \frac{3}{2}(-1\frac{1}{3}) &= -9 \\ -13 + \frac{3}{2}(\frac{4}{3}) &= -14.5 \\ 6 + \frac{3}{2}(\frac{4}{3}) &= 9 \\ -23 + \frac{3}{2}(-1) &= -103/2 \end{aligned}$$

then modify row 6

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 28 \\ 0 & -1 & 9 & 10 & -15 & 2 & -43 \end{bmatrix}$$

→ step 8 [row 5 - (3/3) row 4]

$$\begin{aligned} 2 + \frac{3}{3}(-3) &= 0 \quad - \quad * \\ -4 + \frac{3}{3}(9) &= 2 \quad - \quad * \\ 6 + \frac{3}{3}(-6) &= 0 \quad - \quad * \\ 10 + \frac{3}{3}(-13) &= 4/3 \quad - \quad * \\ 0 + \frac{3}{3}(6) &= 4 \\ 28 + \frac{3}{3}(-23) &= 38/3 \quad - \quad * \end{aligned}$$

→ step 6 [row 3 + 2/3 row 2]

$$\begin{aligned} 2 + \frac{2}{3}(-3) &= 0 \quad - \quad * \\ -1 + \frac{2}{3}(5) &= 7/3 \quad - \quad * \\ -2 + \frac{2}{3}(0) &= -2 \quad - \quad * \\ -1 + \frac{2}{3}(-5) &= -13/3 \\ 2 + \frac{2}{3}(-1) &= 4/3 \\ -19 + \frac{2}{3}(12) &= -11 \end{aligned}$$

→ step 9 [row 6 - (-1/2) row 5]

$$\begin{aligned} -1 + \frac{1}{2}(2) &= 0 \quad - \quad * \\ 9 + \frac{1}{2}(-4) &= 7 \\ -10 + \frac{1}{2}(6) &= -7 \\ -15 + \frac{1}{2}(10) &= -10 \\ 2 + \frac{1}{2}(0) &= 2 \\ -43 + \frac{1}{2}(38) &= -29 \end{aligned}$$

Row modify it

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 14 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{5} & -2 & -\frac{13}{5} & \frac{4}{5} & -11 \\ 0 & 0 & 15 & -9 & -4 & 9 & -103 \\ 0 & 0 & 2 & 2 & \frac{8}{5} & \frac{4}{5} & \frac{38}{5} \\ 0 & 0 & 7 & -7 & -10 & 2 & -29 \end{bmatrix}$$

→ step 10 [row 4 -  $\frac{45}{14}$  row 3]

$$\begin{aligned} 15 - \frac{45}{14}(\frac{7}{5}) &= 0 \text{ ---} \\ -9 - \frac{45}{14}(-2) &= -18\frac{2}{7} \text{ ---} \\ -14.5 - \frac{45}{14}(-\frac{13}{5}) &= -\frac{4}{7} \text{ ---} \\ 9 - \frac{45}{14}(\frac{4}{5}) &= \frac{38}{7} \text{ ---} \\ -103 - \frac{45}{14}(-11) &= -\frac{103}{2} \end{aligned}$$

→ step 11 [row 5 -  $\frac{4}{15}$  row 4]

$$\begin{aligned} 2 - \frac{4}{15}(\frac{38}{7}) &= 0 \text{ ---} \\ 2 - \frac{4}{15}(-9) &= 4.4 \\ \frac{4}{3} - \frac{4}{15}(-14.5) &= \frac{26}{5} \\ 4 - \frac{4}{15}(9) &= \frac{8}{5} \\ \frac{38}{5} - \frac{4}{15}(-\frac{103}{2}) &= 26.4 \end{aligned}$$

→ step 12 [row 6 -  $\frac{7}{2}$  row 5]

$$\begin{aligned} 7 - \frac{7}{2}(2) &= 0 \text{ ---} \\ -7 - \frac{7}{2}(2) &= -14 \\ -10 - \frac{7}{2}(\frac{4}{3}) &= -\frac{44}{3} \\ 2 - \frac{7}{2}(4) &= -12 \\ -29 - \frac{7}{2}(\frac{38}{5}) &= -\frac{220}{5} \end{aligned}$$

Row modify it

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 14 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{5} & -2 & -\frac{13}{5} & \frac{4}{5} & -11 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{38}{7} & -\frac{113}{7} \\ 0 & 0 & 0 & 4.4 & \frac{8}{5} & \frac{26}{5} & 26.4 \\ 0 & 0 & 0 & -14 & -\frac{44}{3} & -12 & -\frac{220}{5} \end{bmatrix}$$

→ step 13 [row 5 +  $\frac{77}{45}$  row 4]

$$\begin{aligned} 4.4 + \frac{77}{45}(-\frac{18}{7}) &= 0 \text{ ---} \\ 26.4 + \frac{77}{45}(-\frac{4}{7}) &= \frac{38}{9} \\ \frac{8}{5} + \frac{77}{45}(\frac{38}{7}) &= \frac{29}{3} \\ 26.4 + \frac{77}{45}(-\frac{113}{7}) &= -\frac{11}{9} \end{aligned}$$

→ step 14 [row 6 +  $\frac{14}{4.4}$  row 5]

$$\begin{aligned} -14 + \frac{14}{4.4} \times 4.4 &= 0 \text{ ---} \\ -\frac{44}{3} + \frac{14}{4.4}(\frac{26}{5}) &= \frac{62}{33} \\ -12 + \frac{14}{4.4}(\frac{8}{5}) &= -\frac{76}{11} \\ -\frac{220}{5} + \frac{14}{4.4}(26.4) &= \frac{52}{3} \end{aligned}$$

Row modify it

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 14 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{5} & -2 & -\frac{13}{5} & \frac{4}{5} & -11 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{38}{7} & -\frac{113}{7} \\ 0 & 0 & 0 & 0 & \frac{29}{3} & \frac{26}{5} & \frac{11}{9} \\ 0 & 0 & 0 & 0 & 0 & \frac{52}{3} & \frac{38}{3} \end{bmatrix}$$



Eliz Blessing OLOKITA  
15/2/2021/013



→ step is  $[ROW6 - \frac{93}{209} ROW5]$   $\frac{38}{9} T_5 + \frac{29}{3} T_6 = -\frac{11}{9}$

$$\frac{62}{3} - \frac{93}{209} \times \frac{38}{9} = 0 \rightarrow$$

Recall that  $T_6 = -1$ .

$$\frac{-76}{11} - \frac{93}{209} \times \frac{29}{3} = -\frac{213}{19}$$

$$\frac{38}{9} T_5 + \frac{29}{3} (-1) = -\frac{11}{9}$$

$$\frac{32}{3} + \frac{93}{209} \times \left(\frac{11}{9}\right) = \frac{213}{19}$$

$$\frac{38}{9} T_5 = -\frac{11}{9} + \frac{29}{3}$$

Then modify it

$$\frac{38}{9} T_5 = \frac{76}{9}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} & -11 \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{33}{7} & -\frac{113}{7} \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{29}{3} & -\frac{11}{9} \\ 0 & 0 & 0 & 0 & 0 & -\frac{213}{19} & \frac{213}{19} \end{bmatrix}$$

$$\frac{9}{38} \times \frac{38}{9} T_5 = \frac{76}{9} \times \frac{9}{38}$$

$$T_5 = 2$$

$$-\frac{18}{7} T_4 + \frac{4}{7} T_5 + \frac{33}{7} T_6 = -\frac{113}{7}$$

$$-\frac{18}{7} T_4 + \frac{4}{7} (2) + \frac{33}{7} (-1) = -\frac{113}{7}$$

The matrix form.

$$-\frac{18}{7} T_4 = -\frac{113}{7} + \frac{8}{7} + \frac{33}{7}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & \frac{7}{3} & -2 & -\frac{13}{3} & \frac{4}{3} \\ 0 & 0 & 0 & -\frac{18}{7} & -\frac{4}{7} & \frac{33}{7} \\ 0 & 0 & 0 & 0 & \frac{38}{9} & \frac{29}{3} \\ 0 & 0 & 0 & 0 & 0 & -\frac{213}{19} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -\frac{113}{7} \\ -\frac{11}{9} \\ \frac{213}{19} \end{bmatrix}$$

$$-\frac{18}{7} T_4 = \frac{72}{7}$$

$$\frac{7}{18} \times \frac{18}{7} T_4 = \frac{72}{7} \times \frac{7}{18}$$

$$T_4 = 4$$

To get  $T_1, T_2, T_3, T_4, T_5$  and  $T_6$ .

$$\frac{7}{3} T_3 - 2 T_4 - \frac{13}{3} T_5 + \frac{4}{3} T_6 = -11$$

$$\frac{7}{3} T_3 - 2(4) - \frac{13}{3}(2) + \frac{4}{3}(-1) = -11$$

$$\frac{7}{3} T_3 = -11 + 8 + \frac{26}{3} + \frac{4}{3}$$

$$T_3 = 3$$

$$-\frac{213}{19} T_6 = \frac{213}{19}$$

$$T_6 = -1$$

$$-36 + 15 - 10 + 1 = 12$$

$$\frac{-3T_2}{-3} = \frac{12 - 6}{-3}$$

$$T_2 = -2$$

$$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$$

$$T_1 + (-2) - 2(-2) + 4 + 3(2) - 1(1) = 4$$

$$T_1 - 2 + 4 + 6 - 1 = 4$$

$$T_1 = 4$$

$$T_1 - 2 - 6 + 4 + 6 + 1 = 4$$