Allen-Oberenger Kingsley
15/ENG03/006
Civil Engineering

$T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4$
$2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20$
$T_1 + 3T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15$
$5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3$
$-3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16$
$4T_2 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27$

[6x6 matrix Transformation]

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ 3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

* Using ② to multiply row 1 and subtract row 2
2-(2)1; -1-(2)(1); 1-(2)(-2); 2-(2)(1); 1-(2)(3); -3-(2)(-1), 20-(2x4)
= 0, -3, 5, 0, -5, 1, 12
* Using ① to multiply row 1 and subtract row 3
1-(1)(1); 3-(1)(1); -3-(1)(-2), -1-(1)(1); 2-(1)(3); 1-(1)(-1); -15-(1)(4)
= 0; 2; -1; 2; -1; 2; -19
* Multiply 5 to row 1 and subtract from row 4
5-(5)(1); 2-(5)(1); -1-(5)(-2); -1-(5)(1); 2-(5)(3); 1-(5)(-1); -3-(5)(4)
= 0; -3; 9; -6; -13; 6; -23
* Multiply (-3) to row 1 and subtract from row 5
-3-(-3)(1); -1-(-3)(1); 2-(-3)(-2); 3-(-3)(1); 1-(-3)(3); 3-(-3)(-1); 16-(-3)(4)
= 0; 2; -4; 6; 10; 0; 28
* Multiply 4 by row 1 and subtract from row 6
4-(4)(1); 3-(4)(1); 1-(4)(-2); -6-(4)(1); -3-(4)(3); -2-(4)(-1); -27-4(4)
= 0; -1; 9; -10; -15; 2; -43

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The matrix turns into

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 9 & -10 & -15 & 2 \end{bmatrix} \cdot \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{bmatrix}$$

* Using a factor of $(-2/3)$ to multiply row 2 and subtract from row 3

$2-(-2/3)(3)$; $-1-(-2/3)(5)$; $-2-(-2/3)(0)$; $-1-(-2/3)(-5)$; $2-(-2/3)(-1)$; $-19-(-2/3)(12)$

$= 0; 2.3333; -2; -4.3333; 1.3333; -11$

* Using a factor of (1) to multiply row 2 and subtract from row 4

$-3-(1)(-3)$; $9-(1)(5)$; $-6-(1)(10)$; $-13-(1)(-5)$; $6-(1)(-1)$; $23-(1)(12)$

$= 0; 4; -6; -8; 7; -35$

* Using a factor of $(-2/3)$ to multiply row 2 and subtract from row 5

$2-(-2/3)(-3)$; $-4-(2/3)(5)$; $6-(-2/3)(0)$; $10-(-2/3)(-5)$; $0-(-2/3)(-1)$; $28-(-2/3)(12)$

$= 0; -0.6666; 6; 6.6666; -0.6666$

* Using a factor of $(1/3)$ to multiply row 2 and subtract from row 6

$-1-(1/3)(-3)$; $9-(1/3)(5)$; $-10-(1/3)(0)$; $-15-(1/3)(-5)$; $2-(1/3)(-1)$; $-43-(1/3)(1)$

$= 0; 7.3333; -10; -15.3333; 2.3333; -47$

The matrix becomes

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & -0.6666 & 6 & 6.6666 & -0.6666 \\ 0 & 0 & 7.3333 & -10 & -13.3333 & 2.3333 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ -47 \end{bmatrix}$$

* Using a factor of 0.2856 to multiply row 3 and subtract row 5

$-0.6666-(-0.2856)(2.3333)$; $6-(-0.2856)(2)$; $6.6666-(0.2856)(-4.3333)$; $-0.6666-(-0.2856)(1.3333)$; $36-(-0.2856)(-11)$.

$= 0; 5.4285; 5.4285; -0.28571; 32.85714$.

* Using a factor of $(7.3333/2.3333)$ to multiply row 1 and subtract row 6

$7.3333-(3.1428)(2.3333)$; $-6-(3.1428)(-2)$; $-13.3333(3.1428)(-4.3333)$;

$2.3333-(3.1428)(1.3333)$; $-47-(3.428)(-11)$

≈ 0; -3.71429, 0.2857, -1.88714; -12.4286

The new matrix becomes

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.571 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 5.4285 & 5.4285 & -0.2857 \\ 0 & 0 & 0 & -3.7142 & 0.2857 & -1.8571 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.1421 \\ 32.8571 \\ -12.4286 \end{bmatrix}$$

* Using a factor of $(5.4285/-2.571)$ to multiply row 4 and subtract from row 5

$5.4285-(-2.1114)(-2.571)$; $5.4285-(2.1114)(-0.5714)$, $-0.2857-(-2.1114)(4.7142)$, $32.8571-(-2.1114)(-16.1429)$

≈ 0; 4.2222; 9.6666; -1.2222

* Using a factor of $(-3.7142/-2.571)$ to multiply row 5 and divide row 6

$-3.9142-(1.4446)(-2.571)$; $0.2857-(1.4446)(-0.5714)$; $-1.8571-(1.4446)(4.7142)$;

$-12.4286-(1.4446)(-16.1429)$

$= 0$; 1.1111; -8.6666; 10.8888

The new matrix becomes

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.5714 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 0 & 4.2222 & 9.6666 \\ 0 & 0 & 0 & 0 & 1.1111 & -8.6666 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ -1.2222 \\ 10.8888 \end{bmatrix}$$

* Using a factor of $(1.1111/4.2222)$ to multiply row 5 and subtract row 6

$1.1111-(0.2631)(4.2222)$; $-8.6666-(0.2631)(9.6666)$; $10.8888-(0.2631)(-1.2222)$

≈ 0, -11.2105; 11.2105

The matrix Becomes

$$\begin{bmatrix} 1 & -1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & -1.3333 \\ 0 & 0 & 0 & -2.5714 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 0 & 4.2222 & 9.6666 \\ 0 & 0 & 0 & 0 & 0 & -11.2105 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ -1.2222 \\ 11.2105 \end{bmatrix}$$

Using back substitution

$$-11.2105 T_6 = 11.2105$$

$$T_6 = \frac{11.2105}{-11.2105} = -1$$

$$4.2222 T_5 + 9.6666 T_6 = -1.2222$$

$$T_5 = \frac{-1.2222 - 9.6666(-1)}{4.2222} = 2$$

$$-2.5714 T_4 - 0.5714 T_5 + 4.7142 T_6 = -16.1429$$

$$T_4 = \frac{-16.1429 - 4.7142(-1) + 0.5714(2)}{-2.5714} = 4$$

$$2.3333 T_3 - 2 T_4 - 4.3333 T_3 + 1.3333 T_6 = -11$$

$$T_3 = \frac{-11 - 1.3333(-1) + 4.3333(2) + 2(4)}{-2.3333} = 3$$

$$\Rightarrow -8 T_2 + 5 T_3 + 6 T_4 - 5 T_5 - 6 = 12$$

$$T_2 = \frac{12 + (-1) + 5(2) - 5(3)}{-3} = -2$$

$$T_1 + T_2 + 2 T_3 + T_4 + 8 T_5 - T_6 = 4$$

$$T_1 = 4 + (-1) - 3(2) - 4 + 2(3) - (-2) = 1$$

Therefore $T_1 = 1$

$T_2 = -2$

$T_3 = 3$

$T_4 = 4$

$T_5 = 2$

$T_6 = -1$