

$$\bar{I}_1 + \bar{I}_2 - 2\bar{I}_3 + \bar{I}_4 + 3\bar{I}_5 - \bar{I}_6 = 1$$

$$2\bar{I}_1 - \bar{I}_2 + \bar{I}_3 + 2\bar{I}_4 + \bar{I}_5 - 3\bar{I}_6 = 20$$

$$\bar{I}_1 + 3\bar{I}_2 - 3\bar{I}_3 - \bar{I}_4 + 2\bar{I}_5 + \bar{I}_6 = -15$$

$$5\bar{I}_1 + 2\bar{I}_2 - \bar{I}_3 - \bar{I}_4 + 2\bar{I}_5 + \bar{I}_6 = -3$$

$$-3\bar{I}_1 - \bar{I}_2 + 2\bar{I}_3 + 3\bar{I}_4 + \bar{I}_5 + 3\bar{I}_6 = 16$$

$$4\bar{I}_1 + 3\bar{I}_2 + \bar{I}_3 - 6\bar{I}_4 - 3\bar{I}_5 - 2\bar{I}_6 = -27$$

Transforming the equations to matrix

$$\begin{array}{l} \text{Row 1} \\ \text{Row 2} \\ \text{Row 3} \\ \text{Row 4} \\ \text{Row 5} \\ \text{Row 6} \end{array} \begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{pmatrix} \begin{pmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \\ \bar{I}_5 \\ \bar{I}_6 \end{pmatrix} = \begin{pmatrix} 1 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{pmatrix}$$

Using a factor of (2) to multiply Row 1 and subtract 2
 $2 \cdot \frac{(2)}{0} 1; -1 - \frac{2}{3}(1); 1 - \frac{2}{5}(2); 2 - \frac{2}{0}(1); 1 - \frac{2}{5}(3); -3 - \frac{2}{1}(-1);$
 $20 - \frac{2}{12}(4)$

Using a factor (1) to multiply Row 1 and subtract Row 3
 $1 - (1)(1); 3 - (1)(1); -3 - (1)(-2); -1 - (1)(1); 2 - (1)(3); 1 - (1)(-1); -15 - (1)(-15)$
 $0 \quad 2 \quad -1 \quad 2 \quad -1 \quad 2 \quad -19$

Using a factor (5) to multiply Row 5 and subtract Row 4
 $5 - (5)(1); 2 - (5)(1); -12 - (5)(2); -1 - (5)(1); 2 - (5)(3); 1 - (5)(-1); -3 - (5)(4)$
 $0 \quad -3 \quad 9 \quad -6 \quad -13 \quad 6 \quad -23$

Using a factor (-3) to multiply Row 1 and subtract Row 5
 $-3 - (-3)(1); -1 - (-3)(1); 2 - (-3)(-2); 3 - (-3)(1); 1 - (-3)(3); 3 - (-3)(-1); 16 - (-3)(-16)$
 $0 \quad 2 \quad -4 \quad 6 \quad 10 \quad 0 \quad 26$

Using a factor (4) to multiply Row 1 and subtract Row 6
 $4 - (4)(1); 3 - (4)(1); 1 - (4)(-2); -6 - (4)(1); -3 - (4)(3); -2 - (4)(-1); -27 - (4)(-27)$
 $0 \quad -1 \quad 9 \quad -10 \quad -15 \quad 2 \quad -43$

$$-27 - (4)(-27)$$

$$-43$$

The matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & 1 & 9 & -10 & -15 & 2 \end{pmatrix} \begin{pmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \bar{r}_4 \\ \bar{r}_5 \\ \bar{r}_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -19 \\ -23 \\ 28 \\ -43 \end{pmatrix}$$

Using a factor of $(-\frac{2}{3})$ to multiply Row 2 and subtract from Row 3
 $2 - (-\frac{2}{3})(3); -1 - (-\frac{2}{3})(5); -2 - (-\frac{2}{3})(0); -1 - (-\frac{2}{3})(-5); 2 - (-\frac{2}{3})(-1); -19 - (-\frac{2}{3})(-1)$
 $0 \quad 2.3333 \quad -2 \quad -4.3333 \quad 1.3333 \quad -11$

Using a factor of (1) to multiply Row 2 and subtract Row 4
 $-3 - (1)(-3); 9 - (1)(5); -6 - (1)(0); -13 - (1)(-5); 6 - (1)(-1); 23 - (1)(12)$
 $0 \quad 4 \quad -6 \quad -8 \quad 7 \quad -35$

Using a factor of $(-\frac{2}{3})$ to multiply Row 2 and subtract Row 5
 $2 - (-\frac{2}{3})(-3); -4 - (-\frac{2}{3})(5); 6 - (-\frac{2}{3})(0); 10 - (-\frac{2}{3})(-5); 0 - (-\frac{2}{3})(-1); 28 - (-\frac{2}{3})(-1)$
 $0 \quad -0.6666 \quad 6 \quad 6.6666 \quad -0.6666 \quad 36$

Using a factor of $(\frac{1}{3})$ to multiply Row 2 and subtract Row 6
 $-1 - (\frac{1}{3})(-3); 9 - (\frac{1}{3})(5); -10 - (\frac{1}{3})(0); -15 - (\frac{1}{3})(-5); 2 - (\frac{1}{3})(1); -43 - (\frac{1}{3})(1)$
 $0 \quad 9.9333 \quad -10 \quad -15.3333 \quad 2.3333 \quad -47$

The new matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 4 & 6 & -8 & 7 \\ 0 & 0 & -0.6666 & 6 & 6.6666 & -0.6666 \\ 0 & 0 & 7.3333 & -10 & -13.3333 & 2.3333 \end{pmatrix} \begin{pmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \bar{r}_3 \\ \bar{r}_4 \\ \bar{r}_5 \\ \bar{r}_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 36 \\ -47 \end{pmatrix}$$

Using a factor of 0.2856 to multiply Row 3 and subtract Row 4
 $-0.6666 - (0.2856)(2.3333); 6 - (0.2856)(-2); 6.6666 - (0.2856)(-4.3333); -0.6666 - (0.2856)(1.3333)$
 $0 \quad 0.2856 \quad 5.4285 \quad 5.4285$

$-0.6666 - (0.2856)(1.3333); 36 - (0.2856)(-11)$
 $-0.2857 \quad 32.8571$

Using a factor of $(7.3333/2.3333)$ to multiply Row 2 and subtract Row 6.

$$7.3333 - (3.1428)(2.3333) = 10 - (3.1428)(-2) = 13.3333$$

$$0 \quad -3.71429 \quad 0.2857$$

$$2.3333 - (3.1428)(1.3333) = -47 - (3.428)(-1) = -12.4285$$

$$-1.85714$$

The new matrix become

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.571 & -6.5714 & 4.7142 \\ 0 & 0 & 0 & 5.4285 & 5.4285 & -0.2857 \\ 0 & 0 & 0 & -3.7142 & 6.2857 & -1.8571 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ 32.8571 \\ -12.4285 \end{pmatrix}$$

Using a factor $(5.4285 / -2.571)$ to multiply Row 4 and subtract from Row 5

$$5.4285 - (-2.1114)(-2.571) = 4.2222$$

$$5.4285 - (-2.1114)(-6.5714) = 9.6666$$

$$-0.2857 - (-2.1114)(4.7142) = 10.2222$$

$$32.8571 - (-2.1114)(-16.1429) = 1.2222$$

Using a factor of $(-3.7142 / -2.571)$ to multiply Row 5 and divide Row 6

$$-3.9142 - (1.4446)(-2.571) = 0.2857$$

$$-0.2857 - (1.4446)(-6.5714) = 9.6666$$

$$1.1111$$

$$-1.8571 - (-1.4446)(4.7142) = 8.6666$$

$$-12.4285 - (1.4446)(16.1429) = 10.8888$$

The new matrix become

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.5714 & -6.5714 & 4.7142 \\ 0 & 0 & 0 & 0 & 4.2222 & 9.6666 \\ 0 & 0 & 0 & 0 & 1.1111 & -8.6666 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ -1.2222 \\ 10.8888 \end{pmatrix}$$

Using a factor of $(1.1111 / 4.2222)$ to multiply Row 5 and subtract Row 6

$$1.1111 - (0.2631)(1.1111) = 0$$

$$9.6666 - (0.2631)(9.6666) = 7.4444$$

$$10.8888 - (0.2631)(10.8888) = 7.1111$$

The new matrix becomes

$$\begin{pmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2.3333 & -2 & -4.3333 & 1.3333 \\ 0 & 0 & 0 & -2.5714 & -0.5714 & 4.7142 \\ 0 & 0 & 0 & 0 & -4.2222 & 9.6666 \\ 0 & 0 & 0 & 0 & 0 & -11.2105 \end{pmatrix} \begin{pmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \bar{T}_3 \\ \bar{T}_4 \\ \bar{T}_5 \\ \bar{T}_6 \end{pmatrix} = \begin{pmatrix} 4 \\ 12 \\ -11 \\ -16.1429 \\ -1.2222 \\ 11.2105 \end{pmatrix}$$

Using backsubstitution

$$-11.2105 \bar{T}_6 = 11.2105$$

$$\bar{T}_6 = \frac{11.2105}{-11.2105} = -1$$

$$4.2222 \bar{T}_5 + 9.6666 \bar{T}_6 = -1.2222$$

$$\bar{T}_5 = \frac{-1.2222 - 9.6666(-1)}{4.2222} = 2$$

$$-2.5714 \bar{T}_4 - 0.5714 \bar{T}_5 + 4.7142 \bar{T}_6 = -16.1429$$

$$-2.5714 \bar{T}_4 - 0.5714(2) + 4.7142(-1) = -16.1429$$

$$\bar{T}_4 = \frac{-16.1429 - 4.7142(-1) + 0.5714 \times 2}{-2.5714} = 4$$

$$2.3333 \bar{T}_3 - 2 \bar{T}_4 - 4.3333 \bar{T}_5 + 1.3333 \bar{T}_6 = -11$$

$$\bar{T}_3 = \frac{-11 - 1.3333(-1) - 4.3333(2) + 2(4)}{2.3333} = 3$$

$$-3 \bar{T}_2 + 5 \bar{T}_3 + 6 \bar{T}_4 - 5 \bar{T}_5 - \bar{T}_6 = 12$$

$$\bar{T}_2 = \frac{12 + (-1) + 5(2) - 5(3) - (-2)}{-3} = -2$$

$$\Rightarrow \bar{T}_1 + \bar{T}_2 - 2 \bar{T}_3 + \bar{T}_4 + 3 \bar{T}_5 - \bar{T}_6 = 4$$

$$\bar{T}_1 = \frac{4 - (-2) - 3(2) - 4 + 2(3) - (-2)}{1} = 1$$

Therefore

$$\bar{T}_1 = 1$$

$$\bar{T}_2 = -2$$

$$\bar{T}_3 = 3$$

$$\bar{T}_4 = 4$$

$$\bar{T}_5 = 2$$

$$\bar{T}_6 = -1$$