

$$\begin{aligned} T_1 + T_2 - 2T_3 + T_4 + 8T_5 - T_6 &= 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 &= 20 \\ T_1 - 8T_2 - 8T_3 - T_4 + 2T_5 + T_6 &= -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 &= -8 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 8T_6 &= 16 \\ 4T_1 + 8T_2 + T_3 - 6T_4 - 3T_5 - 8T_6 &= -22 \end{aligned}$$

corresponding

1	1	-2	1	3	-	1	$T_1$	4
3	-1	1	2	1	-3		$T_2$	20
1	3	3	-1	2	1		$T_3$	-15
5	2	-1	-1	2	1		$T_4$	-8
-3	7	2	3	1	3		$T_5$	16
4	3	1	-1	-3	2		$T_6$	-22

To make  $a_{21} = 0$

Row 2 -  $\left(\frac{a_{21}}{a_{11}}\right)$  Row 1 but  $\frac{a_{21}}{a_{11}} = 2$

For  $a_{21}$  :  $2 - (2)(1) = 0$

$a_{22}$  :  $-1 - (2)(1) = -3$

$a_{23}$  :  $1 - (2)(-2) = 5$

$a_{24}$  :  $2 - (2)(1) = 0$

$a_{25}$  :  $1 - (2)(3) = -5$

$a_{26}$  :  $-3 - (2)(-1) = -1$

$a_{27}$  :  $20 - (2)(4) = 12$

To make  $a_{31} = 0$

Row 3 -  $\left(\frac{a_{31}}{a_{11}}\right)$  Row 1 but  $\frac{a_{31}}{a_{11}} = 1$

For  $a_{31}$  :  $1 - (1)(1) = 0$

$a_{32}$  :  $3 - (1)(1) = 2$

$a_{33}$  :  $-8 - (1)(3) = -11$

$a_{34}$  :  $-1 - (1)(2) = -3$

$a_{35}$  :  $2 - (1)(1) = 1$

$a_{36}$  :  $1 - (1)(-1) = 2$

$a_{37}$  :  $-15 - (1)(4) = -19$

To make  $a_{41} = 0$

Row 4 -  $\left(\frac{a_{41}}{a_{11}}\right)$  Row 1. But  $\frac{a_{41}}{a_{11}} = 5$

For  $a_{41}: 5(-5)1 = 0$

$$a_{42}: 2 - (5)1 = -3$$

$$a_{43}: -1 - (5)(-2) = 9$$

$$a_{44}: -1 - (5)1 = -6$$

$$a_{45}: 7 - (5)3 = -8$$

$$a_{46}: 1 - (5)(0) = -4$$

$$a_{47}: -3 - 5(4) = -23$$

To make  $a_{51} = 0$

Row 5 -  $\left(\frac{a_{51}}{a_{11}}\right)$  Row 1. But  $\frac{a_{51}}{a_{11}} = -3$

For  $a_{51}: -3(-3)1 = 0$

$$a_{52}: -1 - (-3)1 = 2$$

$$a_{53}: 2 - (-3)(-2) = -4$$

$$a_{54}: 3 - (-3)1 = 6$$

$$a_{55}: 1 - (-3)3 = 10$$

$$a_{56}: 3 - (-3)(-1) = 0$$

$$a_{57}: 16 - (-3)4 = 28$$

To make  $a_{61} = 0$

Row 6 -  $\left(\frac{a_{61}}{a_{11}}\right)$  Row 1. But  $\frac{a_{61}}{a_{11}} = 4$

For,

$$a_{61}: 4(-4)1 = 0$$

$$a_{62}: 3 - (4)1 = -1$$

$$a_{63}: 1 - (4)(-2) = 9$$

$$a_{64}: -1 - (4)1 = -5$$

$$a_{65}: -3 - (4)3 = -15$$

$$a_{66}: -2 - (4)(-1) = 2$$

$$a_{67}: -23 - (4)(4) = -43$$

Stage 1 (Forward elimination)

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 2 & -1 & -2 & -1 & 2 & -19 \\ 0 & -3 & 9 & -6 & -13 & 6 & -23 \\ 0 & 2 & -4 & 6 & 10 & 0 & 25 \\ 0 & -1 & 9 & -5 & -15 & 2 & -43 \end{array} \right]$$

To make  $a_{32} = 0$

Row 3 -  $(a_{32}/a_{22})$  Row 2 : But  $a_{32}/a_{22} = -2/3$

$$\text{For } a_{32} : 2 - (-2/3)(-3) = 0$$

$$a_{33} : -1 - (-2/3)(5) = 2/3$$

$$a_{34} : -2 - (-2/3)(-5) = -5$$

$$a_{35} : -2 - (-2/3)(0) = -2$$

$$a_{36} : 2 - (-2/3)(-1) = 4/3$$

$$a_{37} : -14 - (-2/3)(12) = -10$$

To make  $a_{42} = 0$

Row 4 -  $(a_{42}/a_{22})$  Row 2, But  $a_{42}/a_{22} = 1$

$$\text{For } a_{42} : -3 - (1)(-3) = 0$$

$$a_{43} : 9 - (1)(5) = 4$$

$$a_{44} : -6 - (1)(0) = -6$$

$$a_{45} : -13 - (1)(-5) = -8$$

$$a_{46} : 6 - (1)(-1) = 7$$

$$a_{47} : -23 - (1)(12) = -35$$

To make  $a_{52} = 0$

Row 5 -  $(a_{52}/a_{22})$  Row 2 ; But  $a_{52}/a_{22} = -2/3$

$$\text{For } a_{52} : 2 - (-2/3)(-3) = 0$$

$$a_{53} : -4 - (-2/3)(5) = -2/3$$

$$a_{54} : 6 - (-2/3)(-5) = 0$$

$$a_{55} : 10 - (-2/3)(-5) = 20/3$$

$$a_{56} : 0 - (-2/3)(-1) = -2/3$$

$$a_{57} : 23 - (-2/3)(12) = 27$$



To make  $a_{62} = 0$

Row 6  $\leftarrow (a_{62}/a_{22}) \text{ Row 2, But } a_{62}/a_{22} = 1/3$

$$\text{For } a_{62} = -1 - (1/3)(-3) = 0$$

$$a_{63} = 9 - (1/3)(5) = 27/3$$

$$a_{64} = -3 - (1/3)(0) = -3$$

$$a_{65} = -15 - (1/3)(-5) = -40/3$$

$$a_{66} = 2 - (1/3)(-1) = 7/3$$

$$a_{67} = -43 - (1/3)(12) = -47$$

Stage 2

$$\left[ \begin{array}{cccccc|c} 1 & 1 & 2 & 1 & 3 & -1 & 14 \\ 0 & -3 & 5 & 0 & -5 & 4 & 12 \\ 0 & 0 & 2/3 & -2 & -13/3 & 4/3 & -11 \\ 0 & 0 & 4 & -6 & -8 & 2 & -32 \\ 0 & 0 & 4/3 & 6 & -2/3 & -2/3 & 36 \\ 0 & 0 & 23/3 & -5 & -41/3 & 2/3 & 42 \end{array} \right]$$

To make  $a_{45} = 0$

Row 4  $\leftarrow (a_{45}/a_{35}) \text{ Row 3, But } a_{45}/a_{35} = 12/2$

$$\text{For } a_{43} : 4 - (12/2)(1/3) = 0$$

$$a_{44} : -6 - (12/2)(2) = -16/2$$

$$a_{46} : -8 - (12/2)(-2/3) = -4/2$$

$$a_{47} : 7 - (12/2)(4/3) = 33/2$$

$$a_{48} : -35 - (12/2)(-11) = -44/2$$

To make  $a_{53} = 0$

Row 5  $\leftarrow (a_{53}/a_{33}) \text{ Row 3, But } a_{53}/a_{33} = -2/2$

$$\text{For } a_{52} : -2/3 - (-2/2)(2/3) = 0$$

$$a_{54} : 6 - (-2/2)(-2) = 30/4$$

$$a_{55} : 20/3 - (-2/2)(-13/3) = -2/2$$

$$a_{56} : -2/3 - (-2/2)(4/3) = 2/2$$

$$a_{57} : 32 - (-2/2)(-11) = 330/2$$

To make  $a_{63} = 0$

$$\text{Row 6} - \left(\frac{a_{63}}{a_{23}}\right) \text{Row 2; But } \frac{a_{63}}{a_{23}} = \frac{22}{2}$$

$$\text{For } a_{63} = \frac{22}{3} - \left(\frac{22}{2}\right)\left(\frac{2}{2}\right) = 0$$

$$a_{64} = -5 - \left(\frac{22}{2}\right)(-2) = 9/2$$

$$a_{65} = \frac{40}{3} - \left(\frac{22}{2}\right)(-13/3) = 2/2$$

$$a_{66} = 7/3 - \left(\frac{22}{2}\right)(4/2) = -13/2$$

$$a_{67} = 42 - \left(\frac{22}{2}\right)(-11) = -62/2$$

Stage 3 (Forward elimination)

$$\left[ \begin{array}{cccccc|c} 1 & 1 & -2 & 1 & 3 & -1 & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & 12 \\ 0 & 0 & 2/3 & -2 & -5/3 & 4/3 & 11 \\ 0 & 0 & 0 & -18/2 & -4/2 & 83/2 & 73/2 \\ 0 & 0 & 0 & 25/2 & 3/2 & -2/2 & 237/2 \\ 0 & 0 & 0 & 4/2 & 2/2 & -13/2 & -52/2 \end{array} \right]$$

To make  $a_{54} = 0$

$$\text{Row 5} - \left(\frac{a_{54}}{a_{24}}\right) \text{Row 4; But } \frac{a_{54}}{a_{24}} = \frac{-12}{7}$$

$$\text{For } a_{57}: \frac{83}{2} - \left(\frac{-12}{7}\right)\left(\frac{73}{2}\right) = 0$$

$$a_{55}: \frac{83}{2} - \left(\frac{-12}{7}\right)\left(-4/2\right) = 52/7$$

$$a_{56}: -2/7 - \left(\frac{-12}{7}\right)\left(\frac{83}{2}\right) = 224/3$$

$$a_{52}: \frac{237}{2} - \left(\frac{12}{7}\right)\left(\frac{-11}{2}\right) = -11/7$$

To make  $a_{64} = 0$

$$\text{Row 6} - \left(\frac{a_{64}}{a_{24}}\right) \text{Row 4; But } \frac{a_{64}}{a_{24}} = -1/2$$

$$\text{For } a_{67}: 9/2 - (-1/2)\left(\frac{73}{2}\right) = 0$$

$$a_{65}: 2/2 - (-1/2)\left(-4/2\right) = 0$$

$$a_{66}: -13/2 - (-1/2)\left(\frac{83}{2}\right) = 1/2$$

$$a_{62}: \frac{62}{2} - (-1/2)\left(\frac{-13}{2}\right) = -11/2$$

stage 1

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\ 0 & 0 & 2/3 & -2 & 73/3 & 4/3 & : & -11 \\ 0 & 0 & 0 & -13/2 & -4/2 & 33/2 & : & -113/2 \\ 0 & 0 & 0 & 0 & 35/9 & 24/3 & : & -11/9 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & : & -41/12 \end{bmatrix}$$

To make  $a_{65} = 0$

Row 6 - (row 5) Rows. But  $10/12 = 0$

$$\text{for } a_{65} = 0 - (a_{55}) \cdot \frac{24}{9} = 0$$

$$a_{66} = 1/2 - (a_{56}) \left(\frac{24}{9}\right) = 1/2$$

$$a_{67} = 4/2 - 0 \left(-11/2\right) = 4/2$$

stage 2

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 & : & 4 \\ 0 & -3 & 5 & 0 & -5 & -1 & : & 12 \\ 0 & 0 & 2/3 & -2 & 73/3 & 4/3 & : & -11 \\ 0 & 0 & 0 & -13/2 & -4/2 & 33/2 & : & -113/2 \\ 0 & 0 & 0 & 0 & 35/9 & 24/3 & : & -11/9 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & : & -41/12 \end{bmatrix}$$

Back substitution

$$T_6 = 0T_1 + 0T_2 + 0T_3 + 0T_4 + \frac{24}{9}T_5 + \frac{24}{3}T_6 = -11/9$$

$$\frac{38T_5}{9} + \frac{24}{3}T_6 = \frac{-11}{9}$$

$$\frac{38T_5}{9} = \frac{3524}{9}$$

$$T_5 = \frac{3524}{38} = 93.5209$$

$$T_4 : 0T_1 + 0T_2 + 0T_3 + 0T_4 + 0T_5 + 1/2(T_6) = -41/2$$

$$\frac{T_6}{2} = \frac{-41}{2}$$



$$T_4 + 0T_1 + 0T_2 + 0T_3 + (-18/2)T_4 + (-4/2)T_5 + (37/2)T_6 = -113/2$$

$$\frac{-18T_4}{2} - \frac{1018}{19} - \frac{1853}{2} = \frac{-113}{2}$$

$$\frac{-18T_4}{2} = \frac{30672}{153}$$

$$T_4 = -89.6842$$

$$T_3 + 0T_1 + 0T_2 + (2/3)T_3 + (-2)T_4 + (-73/3)T_5 + (4/2)T_6 = 71$$

$$\frac{7T_3}{3} - \left( 2 \times \frac{-1704}{19} \right) - \left( \frac{13}{8} \times \frac{3556}{39} \right) + \left( \frac{4}{3} \times -41 \right) = -11$$

$$= 71$$

$$\frac{7T_3}{3} - \left( 2 \times \frac{-1704}{19} \right) - \left( \frac{13}{8} \times \frac{3556}{39} \right) + \left( \frac{4}{3} \times -41 \right)$$

$$\frac{7T_3}{3} = -11 - \frac{3420}{19} + \frac{4622}{114} + \frac{164}{13}$$

$$T_3 = \frac{15379}{52} \times \frac{3}{2}$$

$$T_3 = 115.6316$$

$$T_2 + 0T_1 + (-3)T_2 + (5)T_3 + (0)T_4 + (-5)T_5 + (7)T_6 = 12$$

$$-3T_2 + 5T_3 - 5T_5 - T_6 = 12$$

$$-3T_2 + \left( 5 \times \frac{46132}{349} \right) - \left( 5 \times \frac{3536}{39} \right) - (-41) = 12$$

$$-3T_2 = 12 - 41 - \frac{250685}{348} + \frac{12780}{38}$$

$$T_2 = \frac{-2646}{19} = -139$$

$$T_6 = 46.4211$$

$$T_1: T_1 + T_2 = (-2T_3) + (1)T_4 + 3(T_5) + (-1)T_6$$

$$T_1 + 88219 + (-2 \times \frac{46132}{349}) + (-17712) + (3 \times \frac{3536}{39}) + (-1 \times -41) = 7$$

$$T_1 = 4 - 41 - \frac{882}{19} + \frac{52224}{322} + \frac{1704}{19} - \frac{12668}{38}$$

$$T_1 = -43.2105$$