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15/ENG 03/014

$$1) \begin{cases} T_1 + T_2 - 2T_3 + T_4 + 3T_5 - T_6 = 4 \\ 2T_1 - T_2 + T_3 + 2T_4 + T_5 - 3T_6 = 20 \\ T_1 + 2T_2 - 3T_3 - T_4 + 2T_5 + T_6 = -15 \\ 5T_1 + 2T_2 - T_3 - T_4 + 2T_5 + T_6 = -3 \\ -3T_1 - T_2 + 2T_3 + 3T_4 + T_5 + 3T_6 = 16 \\ 4T_1 + 3T_2 + T_3 - 6T_4 - 3T_5 - 2T_6 = -27 \end{cases}$$

The Augmented Matrix

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 2 & -1 & 1 & 2 & 1 & -3 \\ 1 & 3 & -3 & -1 & 2 & 1 \\ 5 & 2 & -1 & -1 & 2 & 1 \\ -3 & -1 & 2 & 3 & 1 & 3 \\ 4 & 3 & 1 & -6 & -3 & -2 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} \xrightarrow{\substack{R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \\ R_5 + R_1 \\ R_6 - R_1}} \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 1 & -2 & 3 & 1 & -2 & -4 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 4 & 1 & 1 & -2 & -1 & 2 \\ -2 & 4 & 4 & 4 & 4 & 4 \\ 3 & 2 & 3 & -7 & -6 & -3 \end{bmatrix}$$

$$\xrightarrow{\substack{R_2 \leftrightarrow R_3 \\ R_4 - R_2 \\ R_5 - R_2 \\ R_6 - R_2}} \begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 1 & -2 & 3 & 1 & -2 & -4 \\ 0 & 3 & 5 & -3 & -3 & 6 \\ 0 & 4 & 6 & 6 & 5 & 8 \\ 1 & 4 & 4 & -5 & -8 & -5 \end{bmatrix}$$

$$\xrightarrow{\substack{R_1 - R_2 \\ R_3 + R_2 \\ R_4 - R_2 \\ R_5 - R_2 \\ R_6 - R_2}} \begin{bmatrix} 1 & 0 & -1 & 3 & 2 & -3 \\ 0 & 2 & -1 & -2 & -1 & 2 \\ 1 & 0 & 2 & -1 & -3 & -2 \\ 0 & 1 & 6 & -5 & -4 & 4 \\ 0 & 2 & 8 & 4 & 7 & 2 \\ 0 & 2 & 5 & -8 & -10 & -7 \end{bmatrix}$$

\Rightarrow

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 20 \\ -15 \\ -3 \\ 16 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 12 & -1 & -2 & -1 & 2 \\ 0 & -3 & 9 & -6 & -13 & 6 \\ 0 & 2 & -4 & 6 & 10 & 0 \\ 0 & -1 & 4 & -10 & -15 & 2 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

(1) = Operate eliminate

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -5 & 5 & 0 & -5 & -1 \\ 0 & 2 - (-2/5) & -1 - (-2/5) & -2 - (-2/5) & -1 - (-2/5) & 2 - (-2/5) \\ 0 & -8 - (-2/5) & 9 - (-2/5) & -6 - (-2/5) & -13 - (-2/5) & 6 - (-2/5) \\ 0 & 2 - (-2/5) & -4 - (-2/5) & 6 - (-2/5) & 10 - (-2/5) & 0 - (-2/5) \\ 0 & -1 - (-1/5) & 4 - (-1/5) & -10 - (-1/5) & -15 - (-1/5) & 2 - (-1/5) \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -19 - (-2/5)12 \\ -13 - (-5/5)12 \\ 28 - (-2/5)12 \\ -43 - (-1/5)12 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 24 & 2 & -44 & 14 \\ 0 & 0 & 4 & -6 & -8 & 7 \\ 0 & 0 & (-0.7) & 16 & 6.7 & -0.7 \\ 0 & 0 & (7.4) & (-10) & (-13.4) & 2.4 \end{bmatrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 24 & 2 & -44 & 14 \\ 0 & 0 & 4 - (4/24)24 & -6 - (4/24)24 & -8 - (4/24)24 & 7 - (4/24)24 \\ 0 & 0 & -0.7 - (-0.7/24)24 & 16 - (-0.7/24)24 & 6.7 - (-0.7/24)24 & -0.7 - (-0.7/24)24 \\ 0 & 0 & 7.4 - (-10.7/24)24 & (-10) - (-10.7/24)24 & (-13.4) - (-10.7/24)24 & 2.4 - (-10.7/24)24 \end{bmatrix}$$

(3)

$$\begin{bmatrix} 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -1 \\ 0 & 2 & -1 & -2 & -1 \\ 0 & (-3) & 4 & -6 & -13 \\ 0 & 2 & (-4) & 6 & 10 \\ 0 & (-1) & 4 & (-6) & (-15) \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -14 \\ -23 \\ 24 \\ 43 \end{bmatrix}$$

() = Over to eliminate

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 0 & -5 & 5 & 0 & -5 \\ 0 & 2 - (-2/5) & -1 - (-2/5) & 0 & -5 \\ 0 & -3 - (-2/5) & 4 - (-2/5) & -6 - (-2/5) & -13 \\ 0 & 2 - (-2/5) & -4 - (-2/5) & 6 - (-2/5) & 10 \\ 0 & -1 - (-2/5) & 4 - (-2/5) & -6 - (-2/5) & (-15) \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 0 & -5 & 5 & 0 & -5 \\ 0 & -3 - (-2/5) & 4 - (-2/5) & -6 - (-2/5) & -13 \\ 0 & 2 - (-2/5) & -4 - (-2/5) & 6 - (-2/5) & 10 \\ 0 & -1 - (-2/5) & 4 - (-2/5) & -6 - (-2/5) & (-15) \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -14 \\ -23 \\ 24 \\ 43 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 0 & -3 & 5 & 0 & -5 \\ 0 & 0 & 24 & 2 & -44 \\ 0 & 0 & 4 & -6 & -8 \\ 0 & 0 & (-0.7) & 16 & 6.7 \\ 0 & 0 & 7.4 & (-10) & 13.4 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} 4 \\ 12 \\ -14 \\ -23 \\ 24 \\ 43 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 \\ 0 & -3 & 5 & 0 & -5 \\ 0 & 0 & 24 & 2 & -44 \\ 0 & 0 & 4 & -6 & -8 \\ 0 & 0 & (-0.7) & 16 & 6.7 \\ 0 & 0 & 7.4 & (-10) & 13.4 \end{bmatrix} \begin{matrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{matrix} = \begin{bmatrix} 4 \\ 12 \\ -14 \\ -23 \\ 24 \\ 43 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 31 \\ -37 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 31 \\ -37 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2 & -4 & -4 & 14 \\ 0 & 0 & 0 & -2 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2 & -4 & -4 & 14 \\ 0 & 0 & 0 & -2 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -2 & 1 & 3 & -1 \\ 0 & -3 & 5 & 0 & -5 & -1 \\ 0 & 0 & 2 & -4 & -4 & 14 \\ 0 & 0 & 0 & -2 & -4 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 31 \\ -37 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 31 \\ -37 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -11 \\ -35 \\ 31 \\ -37 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(5)

$$2.4 T_3 - 2 T_2 - 4.4 T_3 + 1.4 T_4 = -11$$

$$2.4 T_3 - 2 (3.977) - 4.4 (2.031) + 1.4 (-0.966) = -11$$

$$2.4 T_3 - 7.954 - 8.9364 - 1.3524 = -11$$

$$2.4 T_3 = -11 + 18.2428$$

$$T_3 = 1.2428 / 2.4$$

$$T_3 = 3.017 \dots (4)$$

$$-3 T_2 + 5 T_3 + 0 T_4 - 5 T_3 - 1 T_4 = 12$$

$$-3 T_2 + 5 (3.017) + 0.5 (2.031) - 1 (-0.966) = 12$$

$$-3 T_2 + 15.085 - 10.155 + 0.966 = 12$$

$$-3 T_2 = 6.104$$

$$T_2 = -6.104 / 3$$

$$T_2 = -2.034 \dots (5)$$

$$T_1 + T_2 - 2 T_3 + T_4 + 3 T_3 - T_4 = 4$$

$$T_1 + (2.034) - 2 (3.017) + 3.977 + 3 (2.031) + 0.966 = 4$$

$$T_1 - 2.034 - 6.034 + 3.977 + 6.093 + 0.966 =$$

$$T_1 = 1.032 \dots (6)$$

From eqn (1) - 1. eqn (6) respectively

$$T_1 = 1.032^\circ\text{C}$$

$$T_2 = -2.034^\circ\text{C}$$

$$T_3 = 3.017^\circ\text{C}$$

$$T_4 = 3.977^\circ\text{C}$$

$$T_5 = 2.031^\circ\text{C}$$

$$T_6 = -0.966^\circ\text{C}$$