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Computer Engineering
15/ENG02/041

ENG382 Assignment 2

① If the maximum percentage absolute error is desired to be 1E-9 using the newton raphson iteration method and initial guess value of 0.5, find the root of the function given in the equation below:

$$f(x) = e^{-0.5x}(4-x) - 2$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$f(x_i) = e^{-0.5x_i}(4 - x_i) - 2$$

$$f'(x_i) =$$

$$u = e^{-0.5x} \qquad v = 4 - x$$

$$du = -0.5e^{-0.5x} \qquad dv = -1$$

$$f'(x_i) = (4-x)(-0.5e^{-0.5x}) - e^{-0.5x}$$

$$\therefore x_{i+1} = x_i - \frac{e^{-0.5x_i}(4-x_i) - 2}{(4-x_i)(-0.5e^{-0.5x_i}) - e^{-0.5x_i}}$$

at iter = 0 $\qquad x = 0.5$

for iter = 1

$$x_i = 0.5$$

$$x_{i+1} = 0.5 - \frac{e^{-0.5(0.5)}(4-0.5) - 2}{(4-0.5)\times(-0.5e^{-0.5\times0.5}) - e^{-0.5\times0.5}}$$

$$x_{i+1} = 0.838890606$$

$$\text{error} = \left| \frac{0.83889606 - 0.5}{0.83889606} \right| \times 100$$

$$= 40.39747328$$

for iter 2

$x_1 = 0.838890606$

$$x_{i+1} = 0.838890606 - \frac{e^{-0.5(0.838890606)}(4-0.838890606) - 2}{(4-0.838890606)\times(-0.5e^{(-0.5\times0.838890606)}) - e^{-0.5\times0.838890606}}$$

$x_{i+1} = 0.884956000$

$$\text{error} = \left|\frac{0.884956000 - 0.838890606}{0.884956000}\right| \times 100$$

~~for iter 3~~

$= 5.205388064\%$

for iter 3

$x_1 = 0.884956000$

$$x_{i+1} = 0.884956000 - \frac{e^{-0.5(0.884956000)}(4-0.884956000) - 2}{(4-0.884956000)\times(-0.5e^{-0.5\times0.884956000}) - e^{-0.5\times0.884956000}}$$

$x_{i+1} = 0.885708605$

$$\text{error} = \left|\frac{0.885708604 - 0.884956000}{0.885708604}\right| \times 100$$

$\text{error} = 0.084971964\%$

for iter 4

$x_1 = 0.885708605$

$$x_{i+1} = 0.885708604 - \frac{e^{-0.5(0.885708604)}(4-0.885708604) - 2}{(4-0.885708604)\times(-0.5e^{-0.5(0.885708604)}) - e^{-0.5(0.885708604)}}$$

$x_{i+1} = 0.885708802$

$$\text{error} = \left| \frac{0.885708802 - 0.885708605}{0.885708802} \right| \times 100$$

$$= 0.000000222\%$$

$$= 2.224207319 \times 10^{-7}\ \%$$

for iter 5

$x_i = 0.885708802$

$$x_{i+1} = 0.885708802 - \frac{e^{-0.5(0.885708802)}(4 - 0.885708802) - 2}{(4 - 0.885708802) \times (-0.5 e^{-0.885708802/2}) - e^{-0.885708802}}$$

$$= 0.885708802$$

$$\text{error} = \left| \frac{0.885708802 - 0.885708802}{0.885708802} \right| \times 100$$

$$= 0$$

iter	x	error (%)
0	0.5	
1	0.838890606	40.39747328
2	0.884956000	5.205388064
3	0.885708605	0.084971964
4	0.885708802	$2.224207319 \times 10^{-7}$
5	0.885708802	0

the root of the equation is 0.885708802.

③ A flat plate of mass m falling in air with velocity v is subjected to a downward gravitational force and an upward frictional drag force due to air. If the drag force, F is given by the equation below

$$F_D = \frac{0.3v^2}{500 + (\ln v)^3} - 0.02x$$

and the terminal velocity is reached when the drag force is equal to gravitational force, that is

$$F_D = mg$$

taking $m = 3.5kg$ and $g = 9.8 m/s^2$. with an initial guess value of $v \approx 18 m/s$.
Using newton raphson's method. find the value of v

Solution

$$F_D = mg$$
$$= 3.5 \times 9.8$$
$$= 34.3$$

$$34.3 = \frac{0.3v^2}{500 + (\ln v)^3} - 0.02x$$

$$f(v) = \frac{0.3v^2}{500 + (\ln v)^3} - 0.02x - 34.3$$

$$v_{i+1} = v_i - \frac{f(v_i)}{f'(v_i)}$$

$$f(v_i) = \frac{0.3v_i^2}{500 + (\ln v_i)^3} - 0.02v_i - 34.3$$

$$f'(v_i) = \frac{d}{dv}\left[\frac{0.3 v_i^2}{500 + (\ln(v_i))^3}\right] - 0.02$$

$$\frac{d}{dv}\left[\frac{0.3 v_i^2}{500 + (\ln(v_i))^3}\right] =$$

$$a = 0.3 v_i^2 \qquad b = 500 + (\ln(v_i))^3$$

$$\frac{da}{dv} = 0.6 v_i^2 \qquad \frac{db}{dv} = 3(\ln(v_i))^2\left(\frac{1}{v}\right)$$

$$= \frac{b\, da/dv - a\, db/dv}{b^2}$$

$$= \frac{\left[500 + (\ln(v_i))^3 (0.6 v_i^2)\right] - \left[0.3 v_i^2 \left((3\ln(v_i))^2 (1/v)\right)\right]}{\left[500 + (\ln(v_i))^3\right]^2} - 0.02$$

$$\therefore v_{i+1} = v_i - \left[\frac{0.3 v_i^2}{500 + (\ln v_i)^3} - 0.02 v_i - 34.3\right]$$

$$\left(\frac{\left[500 + (\ln(v_i))^3 (0.6 v_i^2)\right] - \left[0.3 v_i^2 \left((3\ln(v_i))^2 (1/v)\right)\right]}{\left[500 + (\ln(v_i))^3\right]^2}\right) - 0.02$$

matlab code

```
commandwindow
clear
clc
iter = 0;
v = 18;
for i = 1:inf
    iter(i+1) = i;
```

```
v(i+1)= v(i) -(((0.3*v(i)^2)/(500+(log(v(i))^3))-0.02*(v(i))-
        -34.3)/(((500+(log(v(i))^3)*0.6*(v(i)))-(0.3*(v(i))^2)
        *(3/v(i)*log(v(i)))^2))/(500+(log(v(i)))^3)-0.02));
err(i+1) = abs((v(i+1)-v(i))/v(i+1))*100;
  if   err(i+1)<=1e-5;
     break
  end
  end
b= [iter' v' err']
```

the value of v is 304.067532285 08